A swept-intersection-based remapping method for axisymmetric ReALE computation

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Introduction and motivations

Motivations

- We are interested in the simulation of shock waves interaction in the context of Lagrangian hydrodynamics in axisymmetric coordinates

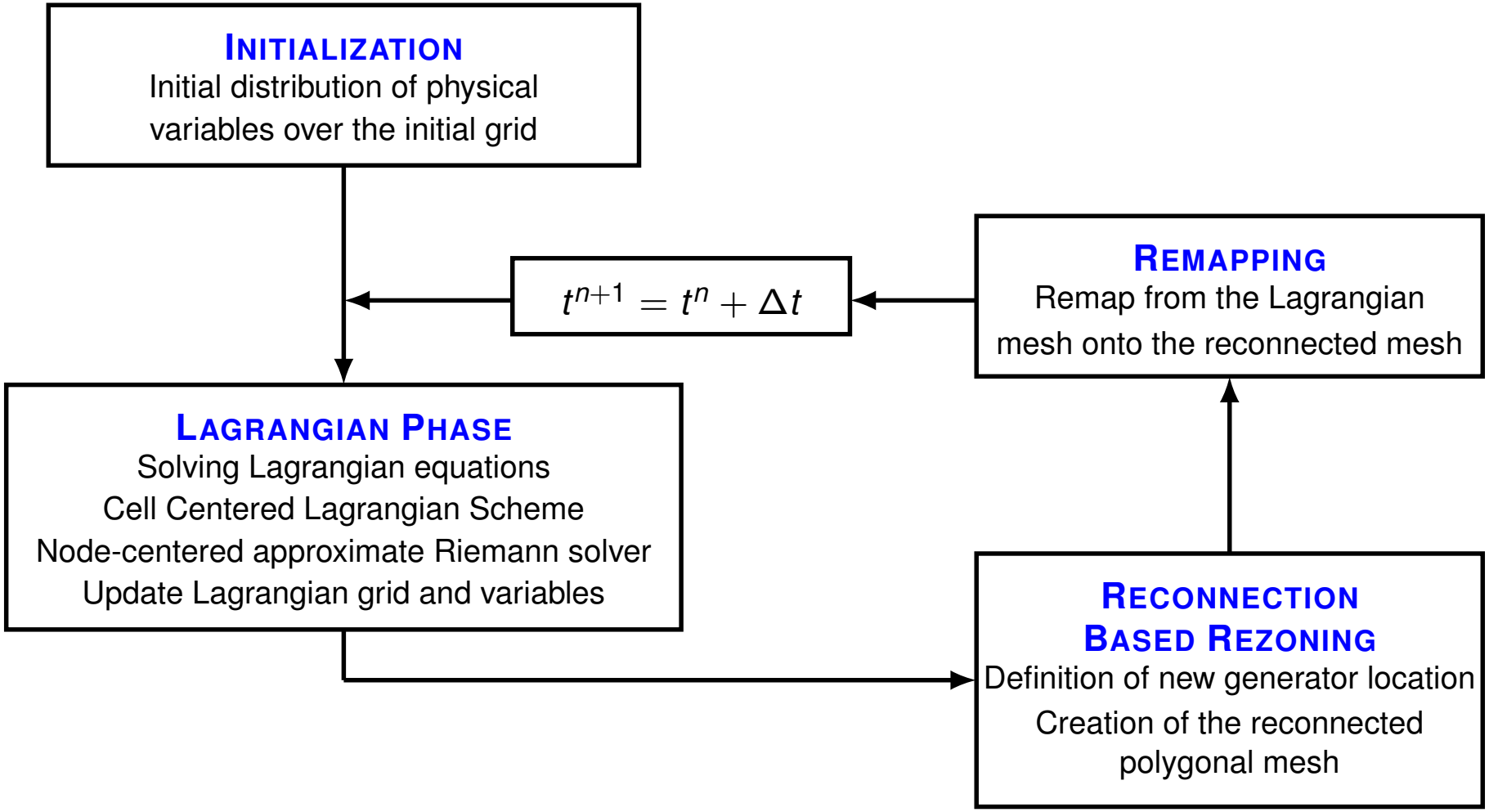
Difficulty

- Pure Lagrangian computation have strong limitation for large deformations
- ALE methods are a solution but still have limitations especially when strong vorticity occurs and when complex geometries are considered

Solution

- The use of a Reconnection based ALE (ReALE) method allow to keep Lagrangian feature in an ALE framework
- We propose here a new remapping method (flux approach) dedicated to ReALE

ReALE Flowchart



Lagrangian phase

Lagrangian Euler equations

For a volume V moving at the velocity $\mathbf{U}$, conservative equations write:

$$\begin{cases} \dfrac{d}{dt}\displaystyle\int_V \rho dV = 0, \\ \dfrac{d}{dt}\displaystyle\int_V dV - \int_V \nabla \cdot \mathbf{U} dV = 0, \\ \dfrac{d}{dt}\displaystyle\int_V \rho \mathbf{U} dV + \int_V \nabla P dV = 0, \\ \dfrac{d}{dt}\displaystyle\int_V \rho E dV + \int_V \nabla \cdot (P\mathbf{U}) dV = 0, \end{cases}$$

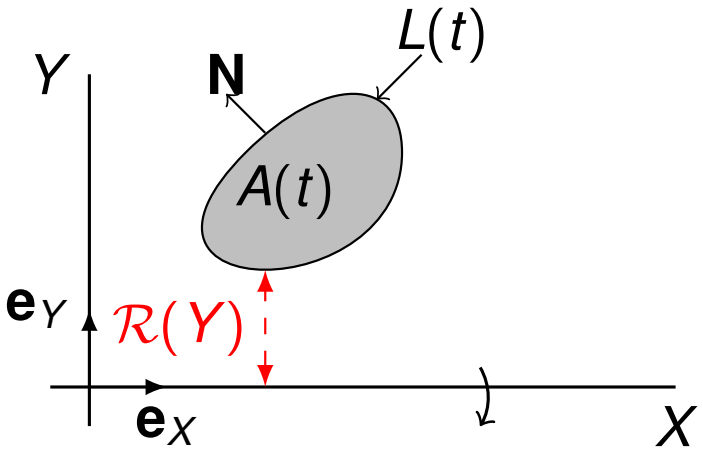
Thermodynamical closures

$$P = P(\rho, \varepsilon) \text{ and } \varepsilon = E - |\mathbf{U}|^2/2$$

Local kinematic equation

$$\frac{d}{dt}\mathbf{X} = \mathbf{U}$$

Area weighted formulation



$$V(t) = \int_{A(t)} \mathcal{R} dA$$

$$S(t) = \int_{L(t)} \mathcal{R} dL$$

$$\mathcal{R}(Y) = Y$$

$$\begin{cases} m\dfrac{d}{dt}\langle \mathbf{U} \rangle + \overline{\mathcal{R}} \displaystyle\int_L P\mathbf{N} dL = 0, \\ m\dfrac{d}{dt}\langle E \rangle + \displaystyle\int_L P\mathbf{N} \cdot \mathcal{R}\mathbf{U} dL = 0, \end{cases}$$

- Averaged values:

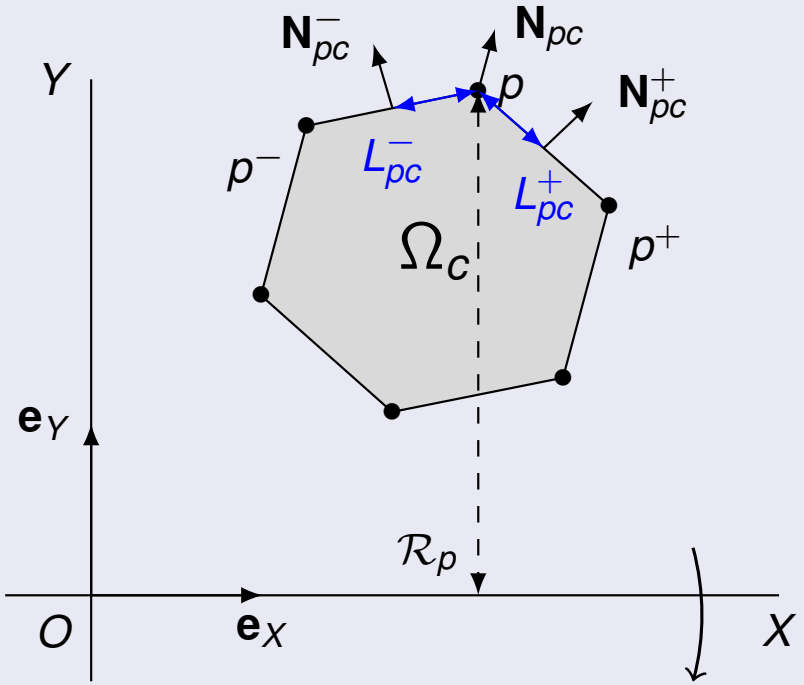
$$\begin{aligned} m &= \int_V \rho dV \\ \langle \phi \rangle &= \frac{1}{m}\int_V \rho\phi dV \\ \overline{\mathcal{R}} &= \frac{V}{A} \end{aligned}$$

M. Billaud Friess, J. Breil, P.-H. Maire, M. Shashkov, *A multi-material CCALE-MOF approach in cylindrical geometry*, CICP accepted.

Area weighted formulation

Cell-centered finite volume scheme

$$\begin{cases} m_c \dfrac{d}{dt}\mathbf{U}_c + \overline{\mathcal{R}}_c \displaystyle\sum_{p\in\mathcal{P}(c)} \mathbf{F}_{pc} = \mathbf{0}, \\ m_c \dfrac{d}{dt}E_c + \displaystyle\sum_{p\in\mathcal{P}(c)} \mathbf{F}_{pc}\cdot\mathcal{R}_p\mathbf{U}_p = 0 \end{cases} \quad \text{and} \quad \frac{d\mathbf{X}_p}{dt} = \mathbf{U}_p, \mathbf{X}_p(0) = \mathbf{X}_p^0$$



$$L_{pc}\mathbf{N}_{pc} = L_{pc}^+\mathbf{N}_{pc}^+ + L_{pc}^-\mathbf{N}_{pc}^-,$$

$$\overline{\mathcal{R}}_c = V_C/A_C \text{ and } \mathcal{R}_p = Y_p$$

$$\mathbf{F}_{pc} = L_{pc}P_c\mathbf{N}_{pc} - \mathrm{M}_{pc}(\mathbf{U}_p - \mathbf{U}_c),$$

$$\mathrm{M}_{pc} = \mathrm{Z}_c\left(L_{pc}^-\mathbf{N}_{pc}^- \otimes \mathbf{N}_{pc}^- + L_{pc}^+\mathbf{N}_{pc}^+ \otimes \mathbf{N}_{pc}^+\right),$$

$$\mathrm{Z}_c = \rho_c a_c,$$

We get $\mathbf{U}_p$ using $\displaystyle\sum_{c\in\mathcal{C}(p)} \mathbf{F}_{pc} = \mathbf{0}$.

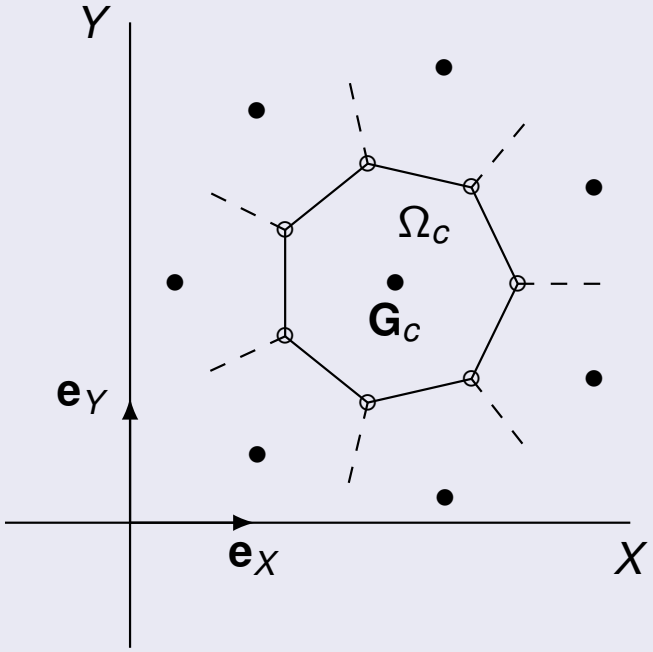
Reconnection-based rezone strategy

Voronoi Tesselation

- Set of generators: $\mathbf{G}_c = (x_c, y_c)$
- Voronoi cell: $\Omega_c = \{\mathbf{X} = (x, y) : |\mathbf{X} - \mathbf{G}_c| < |\mathbf{X} - \mathbf{G}_j|, \text{ for all } j \neq c\}$
- $\mathbf{G}_c^{n+1}$ is calcuted using a convex combination between the new Lagrangian-like position $\mathbf{G}_c^{n+1,lag}$ and the centroid $\mathbf{X}_c^{n+1}$ of the Lagrangian cell at time t^{n+1}:

$$\mathbf{G}_c^{n+1} = \mathbf{G}_c^{n+1,lag} + \omega_c \left(\mathbf{X}_c^{n+1} - \mathbf{G}_c^{n+1,lag} \right)$$

$\omega_c \in [0; 1]$ is constructed using invariants of the right Cauchy Green strain tensor.



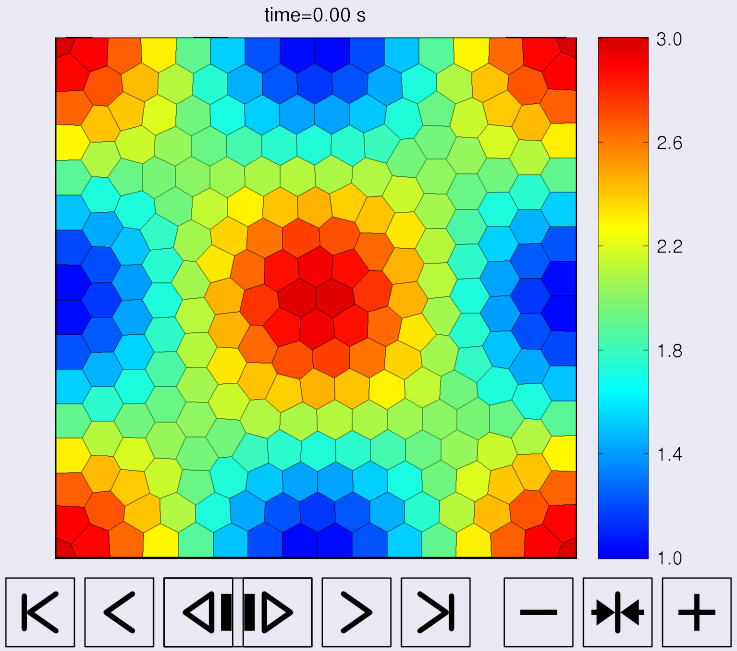
R. Loubère, P.H. Maire, M. Shashkov, J. Breil, S. Galera, *ReALE: A reconnection-based arbitrary-LagrangianEulerian method*, JCP, **229**, 47244761 (2010).

Remapping numerical test: periodic vortex

Description

- Usually a cell-intersection method is used, here we have delopped a method based on a flux approach
- We consider a $[0,1]^2$ domain in which we initialize the density field using $\Psi(x,y) = 2 + \cos(2\pi x)\sin(2\pi y)$.
- We impose that the generators move under the nonlinear unsteady velocity field defined by the following stream function ($T = 3$):

$$\mathcal{F} = \frac{1}{4\pi}\sin^2(\pi x)\sin^2(\pi y)\cos\left(\frac{\pi t}{T}\right)$$



Grid	L_1^{exact}	L_1^{swept}	Order	t^{exact} (s)	t^{swept} (s)
8^2	1.07×10^{-1}	1.07×10^{-1}	-	7.7×10^{0}	3.5×10^{0}
16^2	2.47×10^{-2}	2.48×10^{-2}	2.11	5.3×10^{1}	1.3×10^{1}
32^2	5.15×10^{-3}	5.18×10^{-3}	2.26	4.8×10^{2}	9.1×10^{1}
64^2	1.10×10^{-3}	1.11×10^{-3}	2.22	4.6×10^{3}	8.3×10^{2}

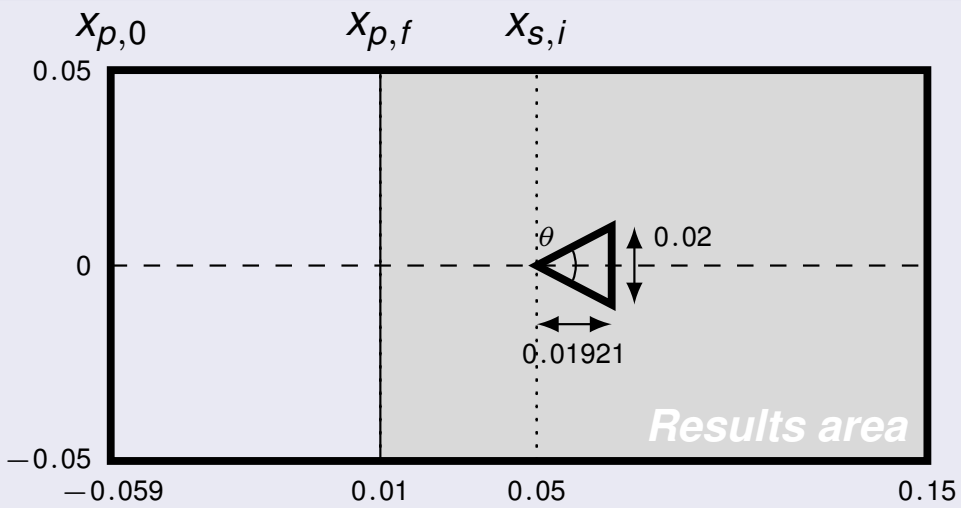
Comparison of relative L_1 error and CPU time t between exact intersection and our flux method

Schardin test case: Presentation

Description

Schardin test case deals with shock wave diffraction on a triangular body. This shock generates curved reflected and diffracted waves.

Configuration

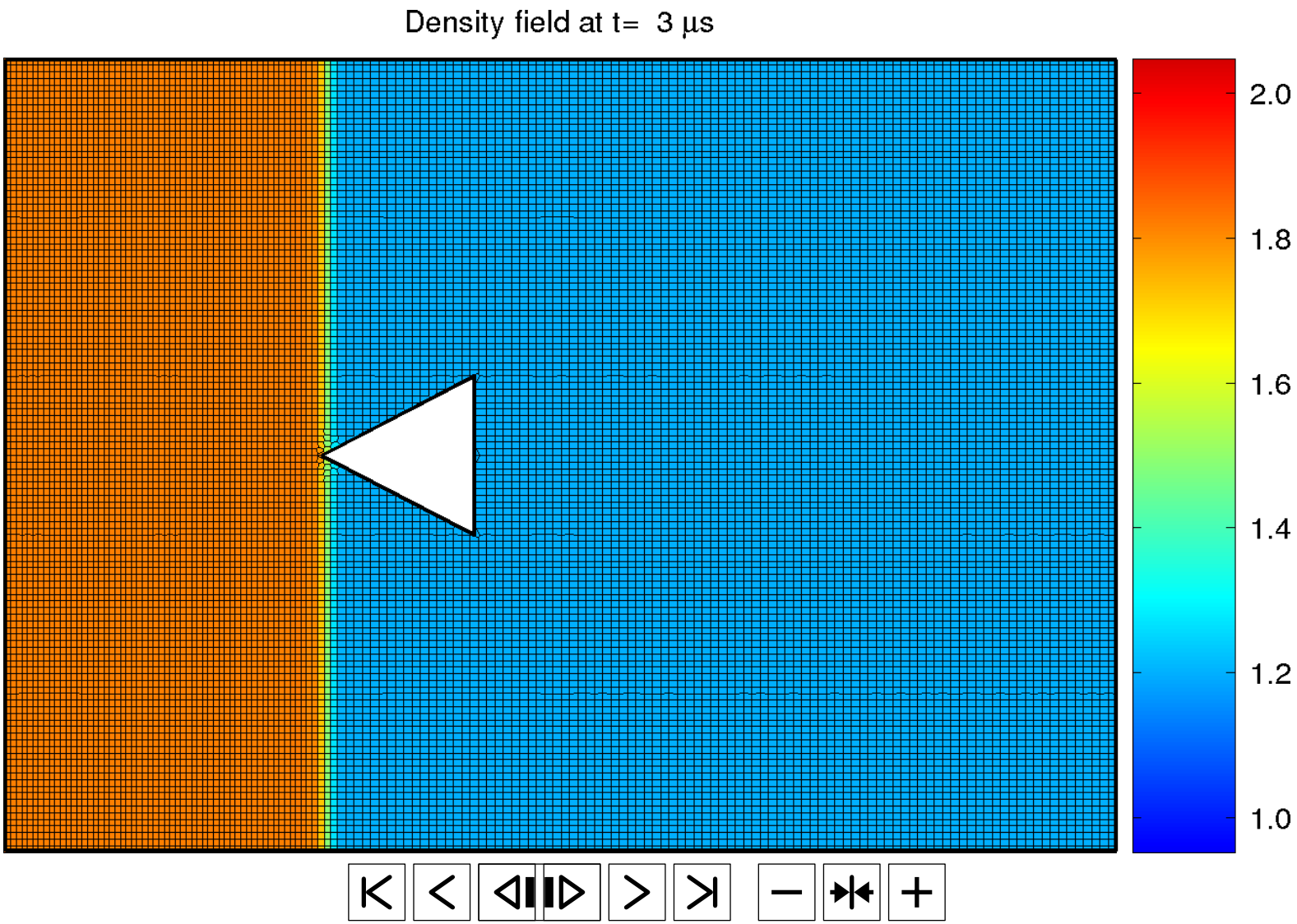


$x_{p,0}$: the initial lagrangian position of the piston
$x_{p,f}$: the final position of the piston
$x_{s,i}$: the position of the incident shock at $t = 0$

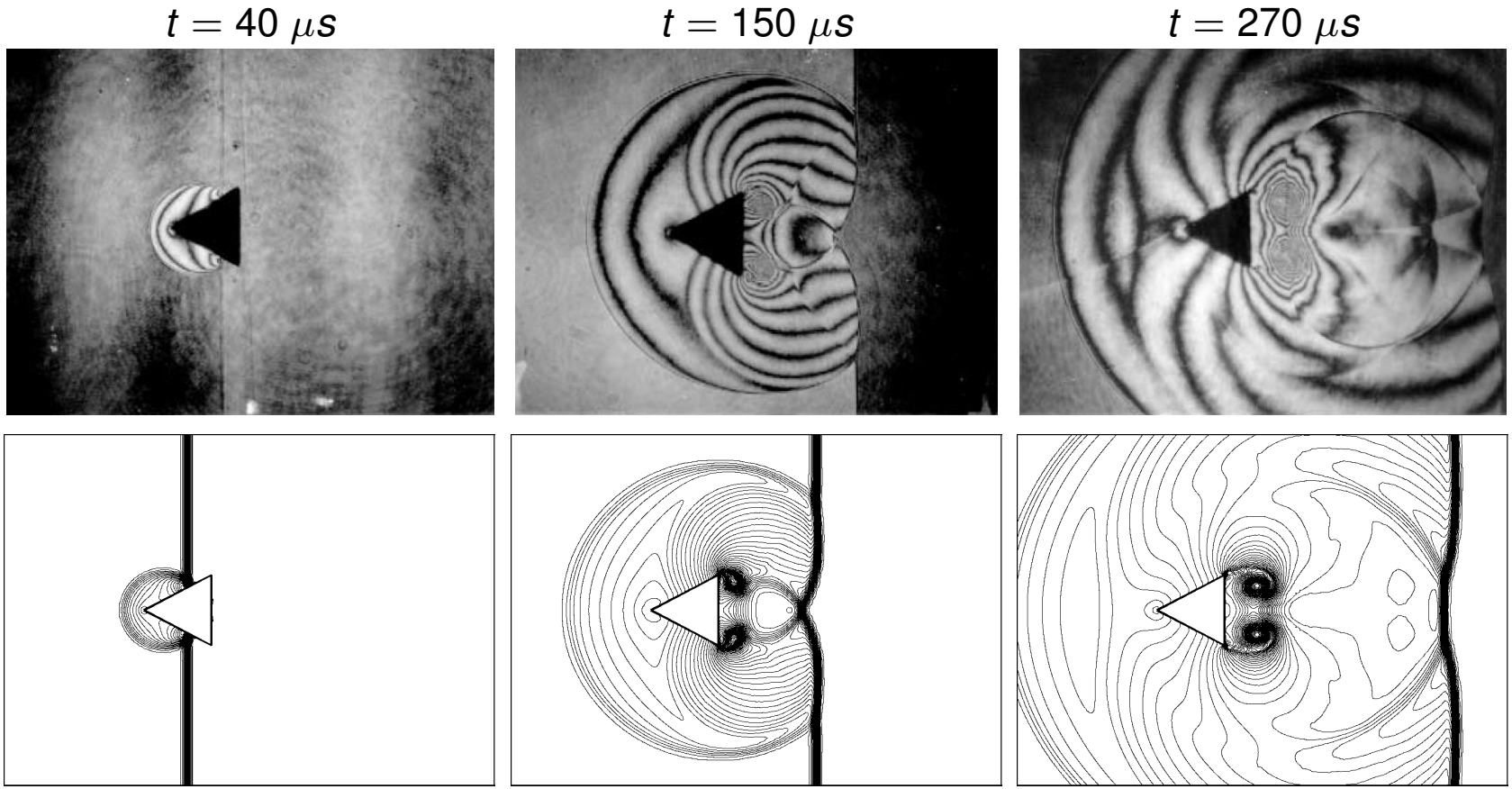
- $M_s = 1.3$
- $P_0 = 0.05\ MPa$
- $\rho_0 = 1.2\ kg.m^{-3}$
- $\gamma = 1.4$
- $\theta = 55°$
- Cell number: 23892
- $t_{piston} = 343\mu s$
- $t_{interaction} = 305\mu s$

T. Harribey, J. Breil, P.-H. Maire, M. Shashkov, *A swept-intersection-based remapping method in a ReALE framework*, IJNMF **72**, pp 697-708 (2013).

Schardin test case: Numerical simulation



Schardin test case: Experimental validation



Comparison between experimental shadowgraphs and computed isopycnics at different times

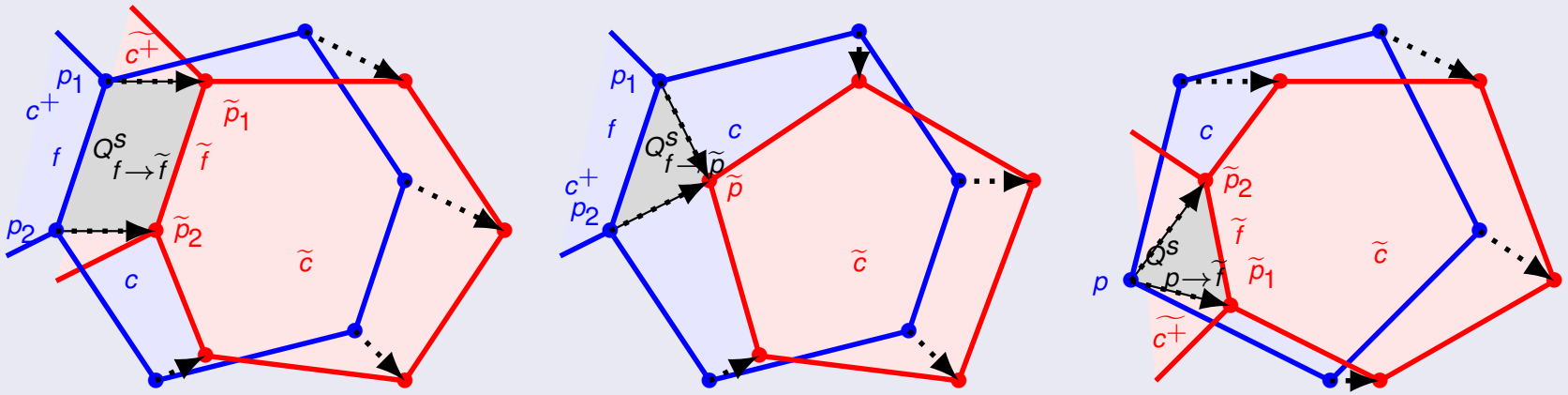
H. SCHARDIN, *High frequency cinematography in the shock tube*, J. Phot. Sci., **5**, 17-19 (1957).

Remapping phase

Main goal of remapping

- Remap the value of $\psi = (\rho, \rho\mathbf{U}, \rho E)$ on the new voronoi mesh to get $\widetilde{\psi}$
- Classical approach of swept fluxes remapping can be used except when change of topology occur

Swept-intersection-based method

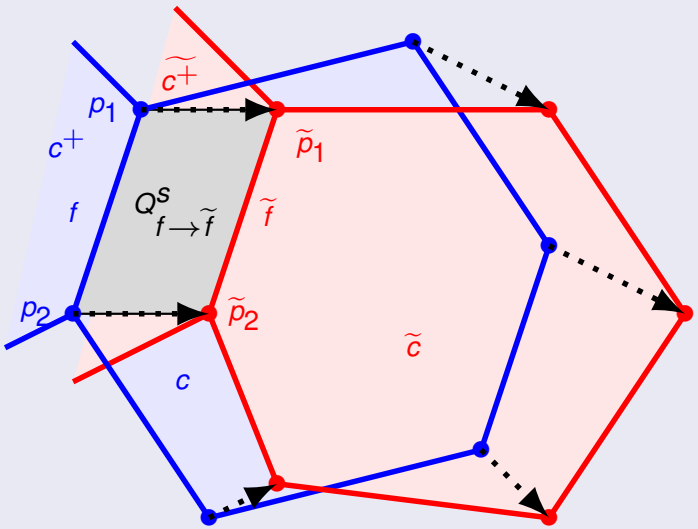


face to face | face to point | point to face

M. Kucharik, M. Shashkov, *Extension of efficient, swept-integration-based conservative remapping method for meshes with changing connectivity*, Int. J. Numer. Meth. Fluids, **56**, 1359-1365 (2008).

Remapping phase

Face to face projection



$$Q^s_{f\to\widetilde{f}} = \{p1, \widetilde{p}_1, \widetilde{p}_2, p_2\}$$

(old) Lagrangian mesh

(new) Reconnected mesh

Given Ψ a physical variable defined on the domain $\{c\}$, its expression in cell $\widetilde{c}$ is:

$$\Psi_{\widetilde{c}} = \frac{1}{V_{\widetilde{c}}} \int_{\widetilde{c}} \mathcal{R}\Psi dX \text{ where } V_{\widetilde{c}} = V_c + \sum_{f\in c} \int_{Q^s_{f\to\widetilde{f}}} \mathcal{R} d\mathbf{X}$$

If $\widetilde{c}^+$ denotes the cell neighbor with $\widetilde{c}$:

$$\Psi_{\widetilde{c}} = \frac{1}{V_{\widetilde{c}}} \left(\int_c \mathcal{R}\Psi d\mathbf{X} + \sum_{f\in c} \int_{Q^s_{f\to\widetilde{f}}} \mathcal{R}\Psi d\mathbf{X} \right)$$

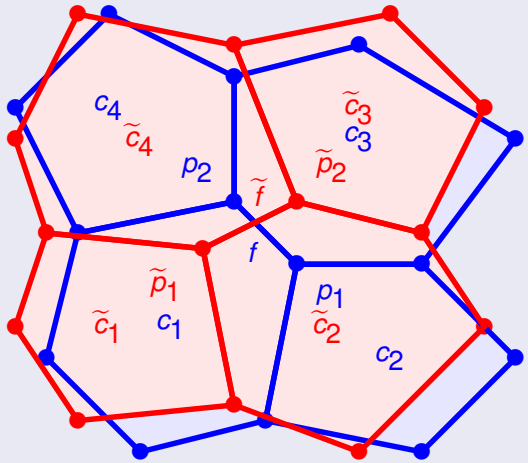
$$\Psi_{\widetilde{c}^+} = \frac{1}{V_{\widetilde{c}^+}} \left(\int_{c^+} \mathcal{R}\Psi d\mathbf{X} - \sum_{f\in c} \int_{Q^s_{f\to\widetilde{f}}} \mathcal{R}\Psi d\mathbf{X} \right)$$

Upwind approximation:

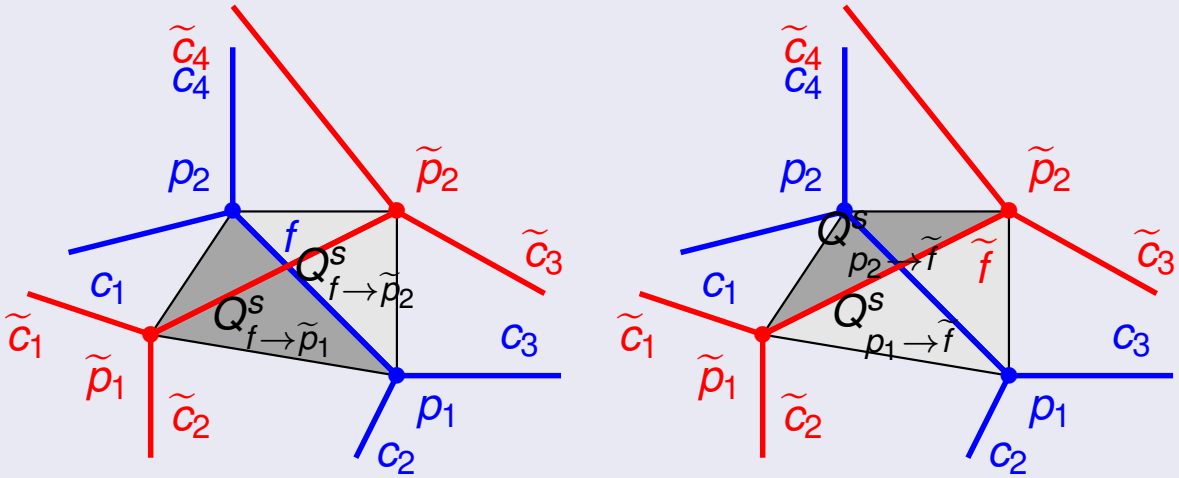
$$\int_{Q^s_{f\to\widetilde{f}}} \mathcal{R}\Psi d\mathbf{X} = \begin{cases} \int_{Q^s_{f\to\widetilde{f}}} \mathcal{R}\Psi_{c^+}(\mathbf{X}) d\mathbf{X} & \text{if } Q^s_{f\to\widetilde{f}} \geq 0 \\ \int_{Q^s_{f\to\widetilde{f}}} \mathcal{R}\Psi_{c}(\mathbf{X}) d\mathbf{X} & \text{otherwise} \end{cases}$$

Voronoi-like reconnection

Illustration



Two projection steps



Flux estimation

If $Q^s_{f\to\widetilde{p}_1} > Q^s_{f\to\widetilde{p}_2}$ then $c_{ref} = c_1$, else $c_{ref} = c_3$. Then we have:

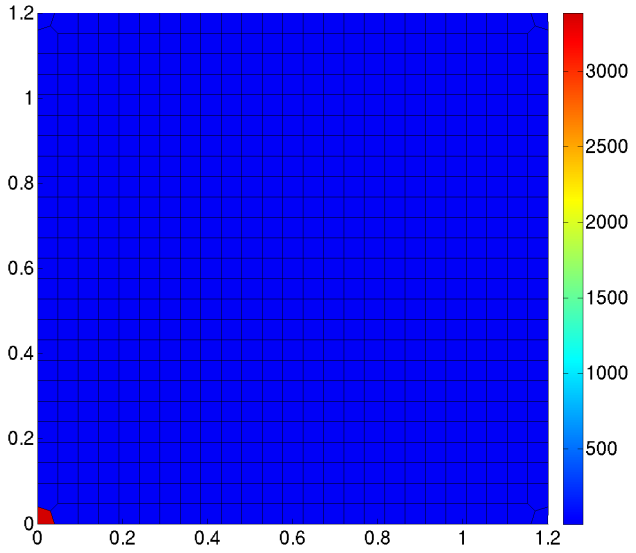
$$\Psi_{\widetilde{c_1}} = \frac{1}{V_{\widetilde{c_1}}}\left(\int_{c_1} \mathcal{R}\Psi_{c_1}\, d\mathbf{X} + \int_{Q^s_{f\to\widetilde{p}_1}} \mathcal{R}\Psi_{c_{ref}}(\mathbf{X})d\mathbf{X}\right) \quad \Psi_{\widetilde{c_3}} = \frac{1}{V_{\widetilde{c_3}}}\left(\int_{c_3} \mathcal{R}\Psi_{c_3}\, d\mathbf{X} - \int_{Q^s_{f\to\widetilde{p}_2}} \mathcal{R}\Psi_{c_{ref}}(\mathbf{X})d\mathbf{X}\right)$$

$$\Psi_{\widetilde{c_2}} = \frac{1}{V_{\widetilde{c_2}}}\left(\int_{c_2} \mathcal{R}\Psi_{c_2}\, d\mathbf{X} - \int_{Q^s_{p_1\to\widetilde{f}}} \mathcal{R}\Psi_{c_{ref}}(\mathbf{X})d\mathbf{X}\right) \quad \Psi_{\widetilde{c_4}} = \frac{1}{V_{\widetilde{c_4}}}\left(\int_{c_4} \mathcal{R}\Psi_{c_4}\, d\mathbf{X} + \int_{Q^s_{p_2\to\widetilde{f}}} \mathcal{R}\Psi_{c_{ref}}(\mathbf{X})d\mathbf{X}\right)$$

Sedov test case

Description

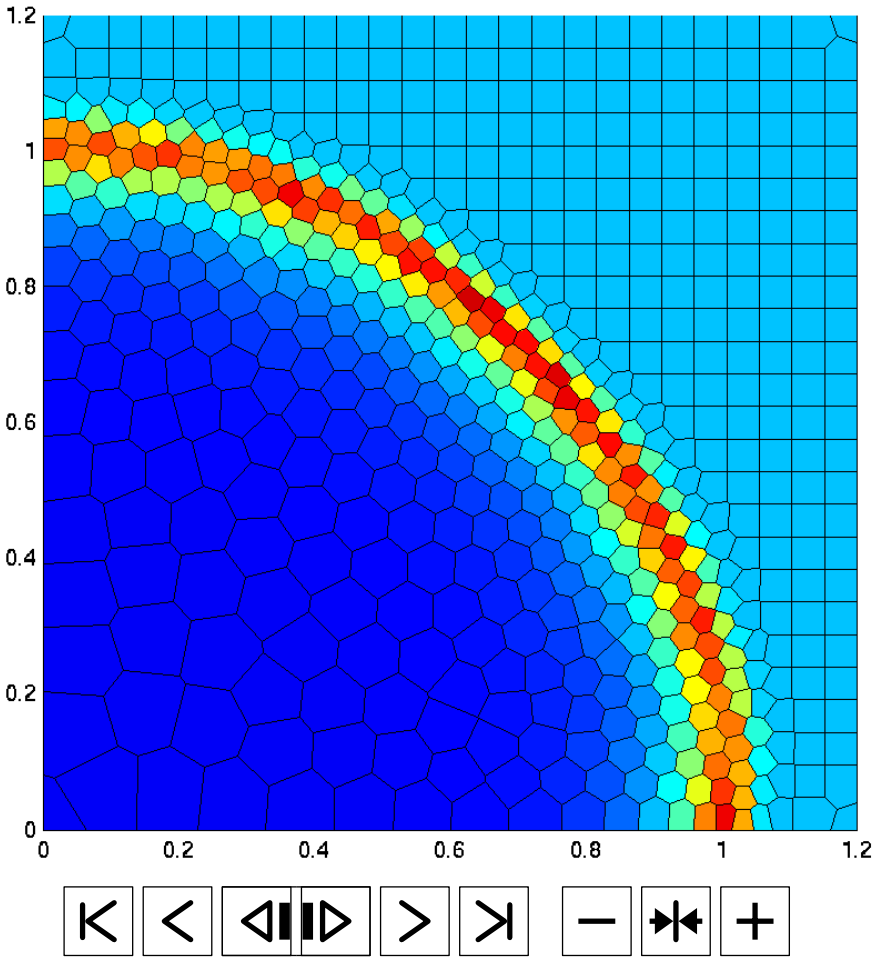
Sedov problem for a point blast in a uniform medium with spherical symmetry



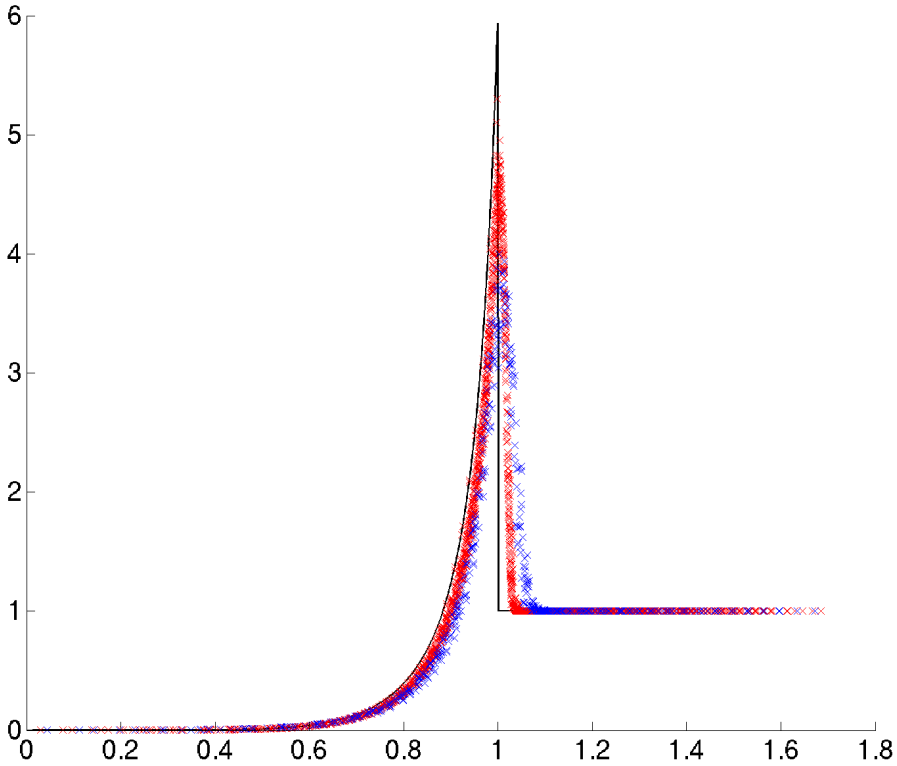
Initial mesh

- Uniform initial condition: $(\rho^0, P^0, \mathbf{U}^0) = (1, 10^{-6}, \mathbf{0})$
- Unstructured mesh: 625 cells
- Imposed pressure at the origin: $P_{or} = (\gamma - 1)\rho_{or}\dfrac{\mathcal{E}_0}{V_{or}}$ with $\mathcal{E}_0 = 0.851072$
- EOS: perfect gaz with $\gamma = 1.4$
- Final time: $t_f = 1$

Sedov test case



Mesh and density

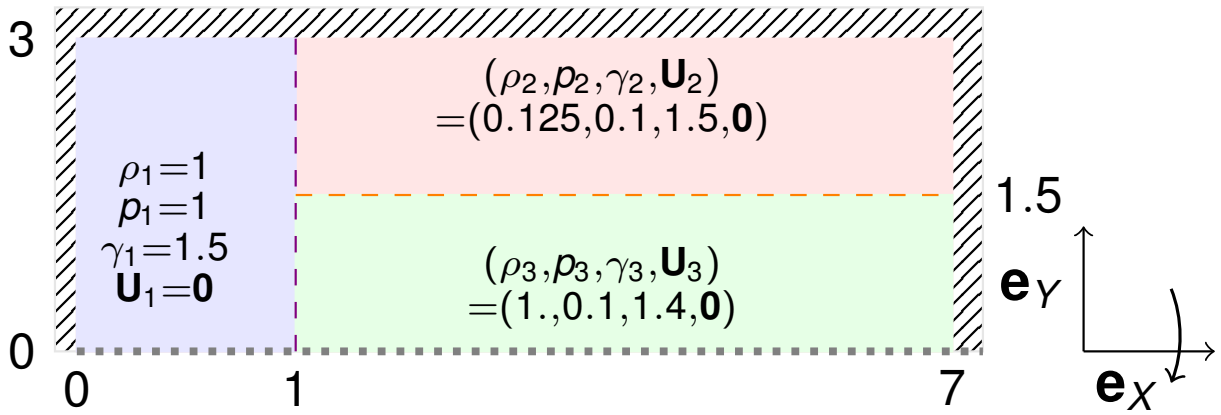


Density profil as a function of cell radius:
numerical ($\times$ 25x25, $\times$ 50x50) vs. exact solution ($-$)

Triple point problem

Description

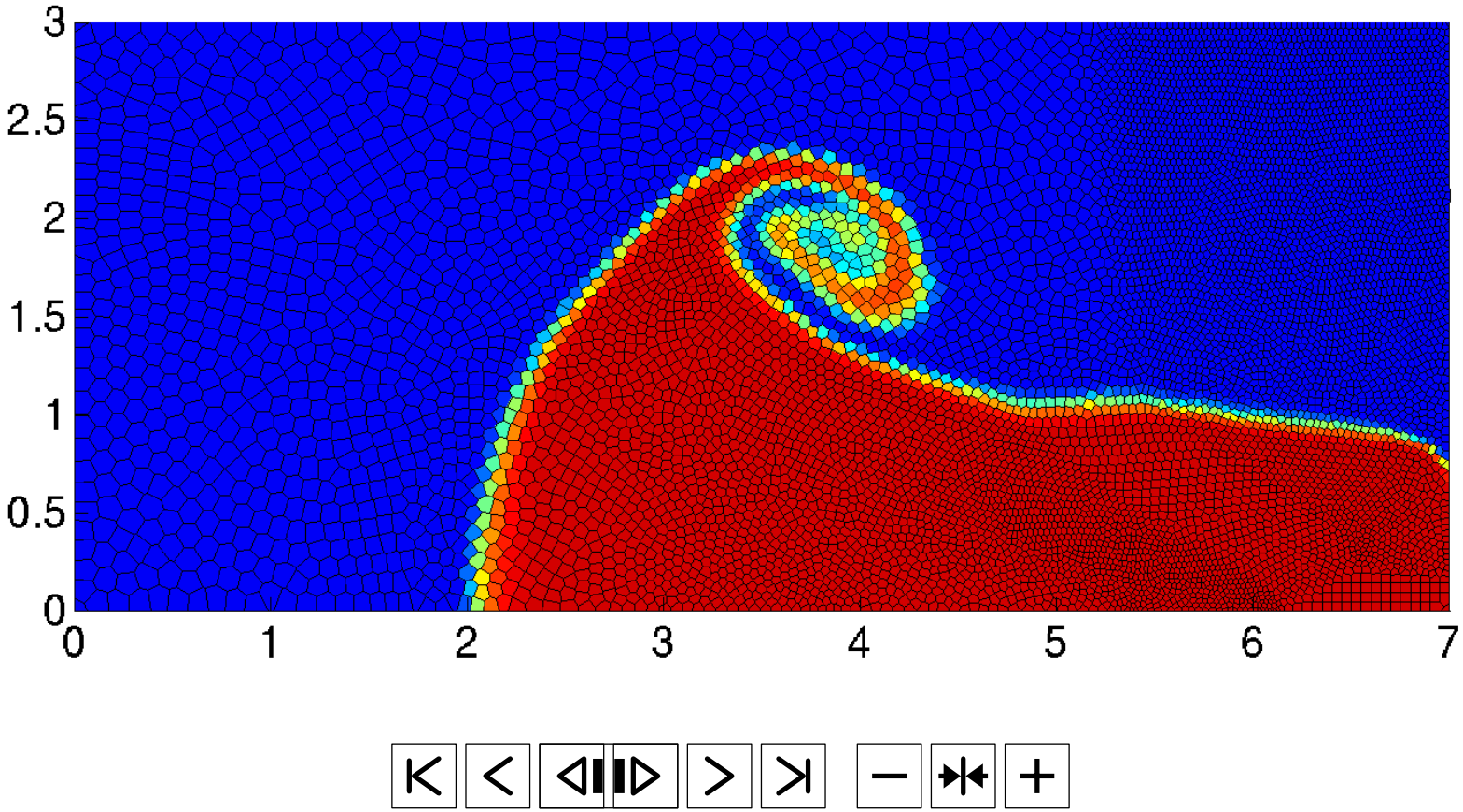
Two-materials problem corresponding to a three-states Riemann problem in an axisymmetric geometry



- Final time: $t_f = 5$
- Cartesian grid: 140×60 cells
- EOS: Perfect gases
- Use mass fraction to follow materials

Triple point problem

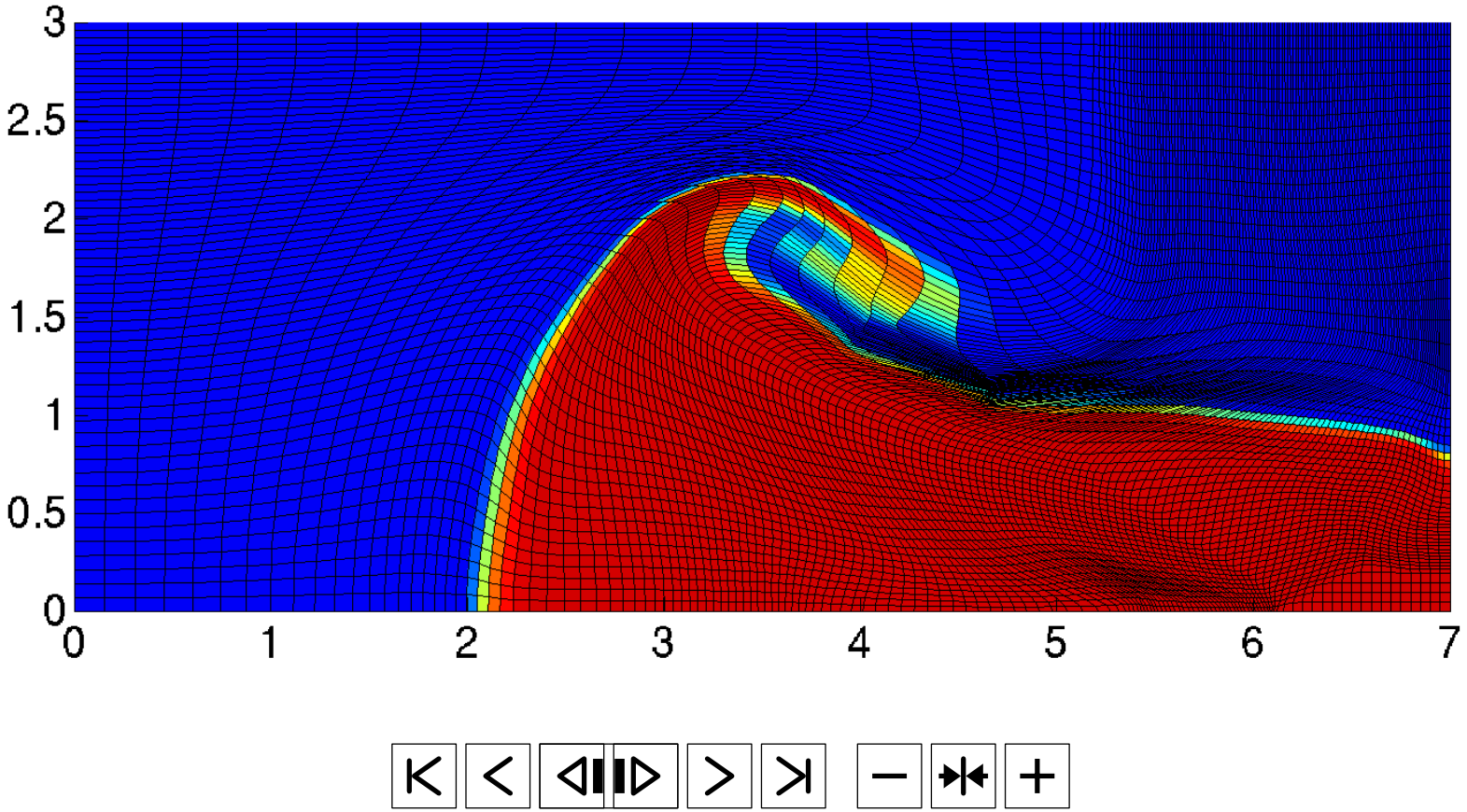
ReALE computation



Mesh and mass fraction evolution

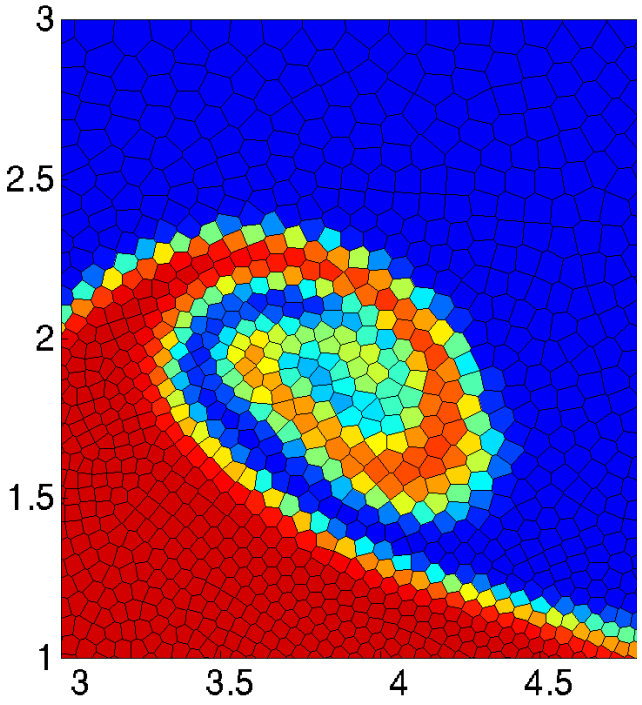
Triple point problem

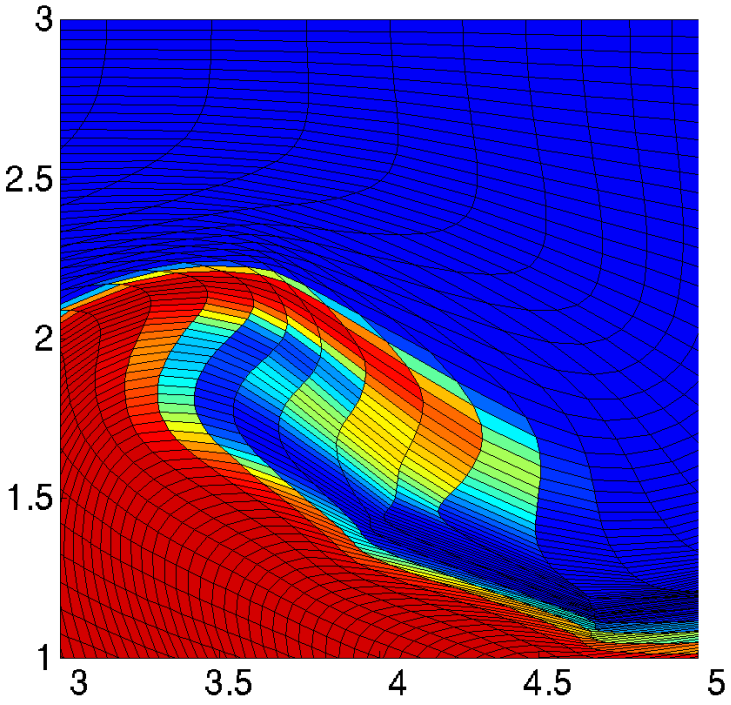
ALE computation



Mesh and mass fraction evolution

Triple point problem





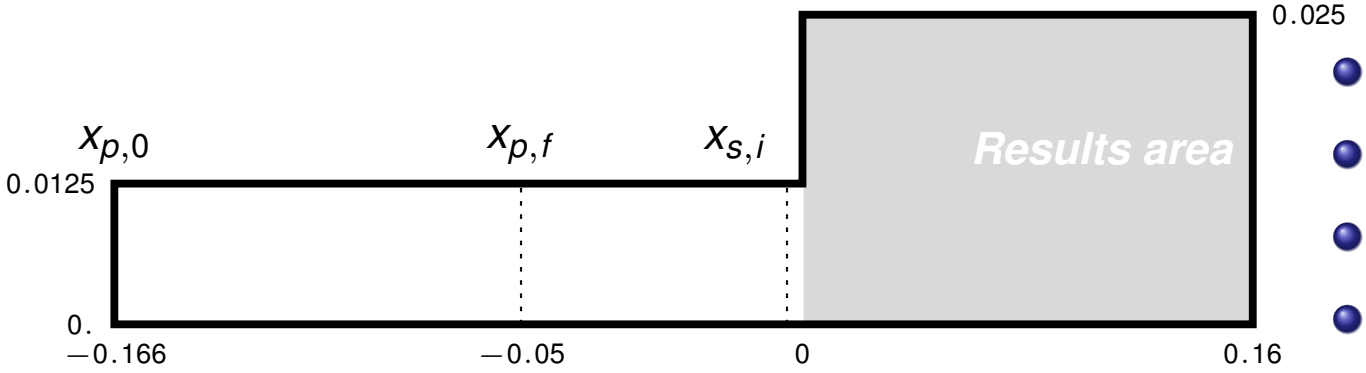
Computational time (s)

Type	Exact	Swept	ratio
ReALE	2.45×10^4	4.15×10^3	6
ALE	8.27×10^3	1.8×10^3	4.6
ratio	3	2.3	13.6

Transient shock wave in tubes: Presentation

Description

Transient shock wave in tubes test case deals with the interaction of shock waves within circular tubes with a sudden area change in cross section.



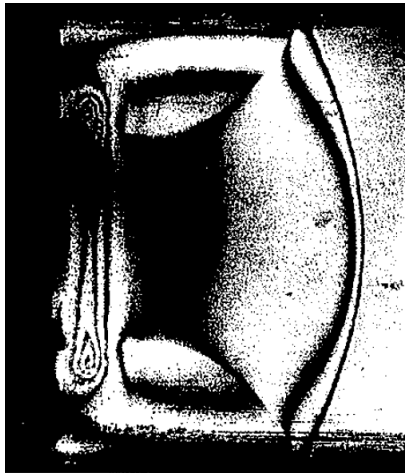
$x_{p,0}$: the initial lagrangian position of the piston
$x_{p,f}$: the final position of the piston
$x_{s,i}$: the position of the incident shock at $t = 0$ with $x_{s,i} = -0.016\ m$

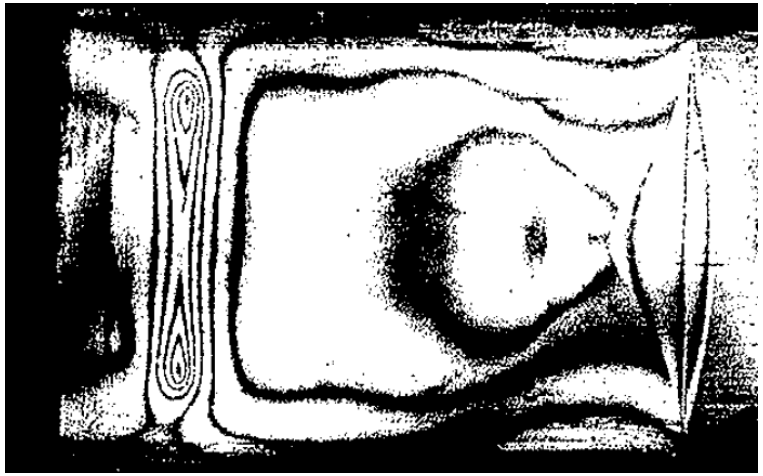
- $M_s = 1.3$, $M_s = 1.5$
- $P_0 = 98285\ Pa$
- $\rho_0 = 1.2\ kg.m^{-3}$
- $\gamma = 1.4$
- Cell number: 24264
- $t_{interaction} = 420 \mu s$

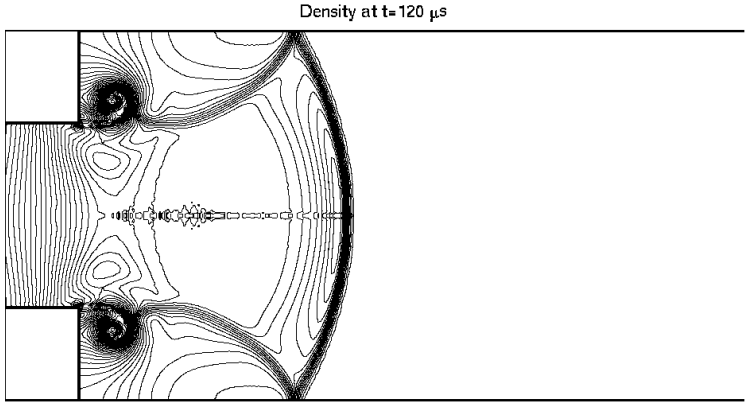
Z. JIANG, K. TAKAYAMA, H. BABINSKY, T. MEGURO *Transient shock wave flows in tubes with a sudden change in cross section*, Shock Waves, **7**: 151-162 (1997).

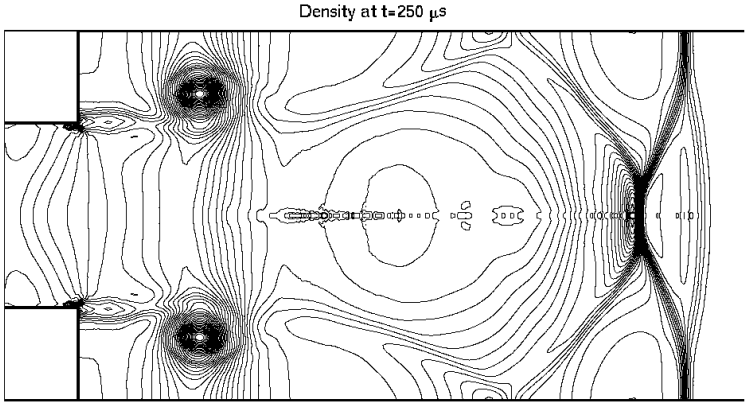
Transient shock wave in tubes $M_s = 1.3$

Transient shock wave in tubes $M_s = 1.3$: Experimental validation





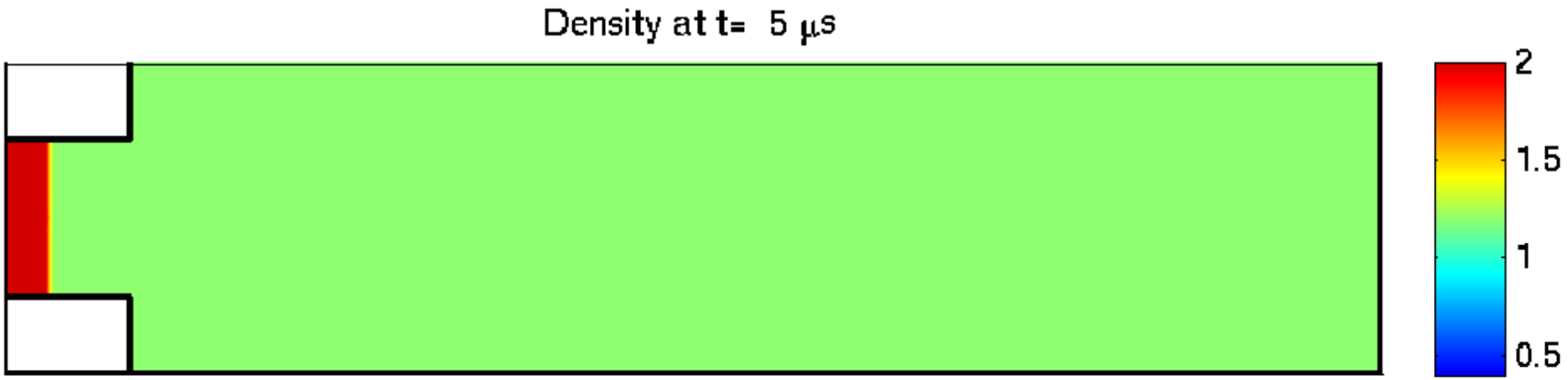




Comparison between experimental interferogram and computed isopycnics at different times

Z. JIANG, K. TAKAYAMA, H. BABINSKY, T. MEGURO *Transient shock wave flows in tubes with a sudden change in cross section*, Shock Waves, **7**: 151-162 (1997).

Transient shock wave in tubes $M_s = 1.5$



Conclusions and perspectives

Summary

- An accurate reconnection-based ALE technique
- Validation of the new swept-intersection-method dedicated to the remapping on several hydrodynamic simulations
- 5 times faster than the classical cell-intersection method with the same accuracy

Future works

- Adapt this method to multi-material simulations (hybrid remapping)
- Add/remove generators dynamically (AMR-like)
- Validate this approach on laser applications
- Using ReALE in Combination with Lagrangian and ALE Methods