A high order cell centred Lagrangian Godunov scheme for elastoplastic flow

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Introduction (1)

- The staggered grid schemes employed in most hydrocodes have been remarkably successful.
- However, they clearly have some theoretical and practical deficiencies.
- High resolution cell centred Lagrangian Godunov schemes could overcome some of these problems.
- However, while Eulerian Godunov methods have been well established for a long time [Godunov (MS, 1959)] progress has been slow in extending these ideas to Lagrangian and ALE schemes.
- This has largely been due to the difficulty in defining consistent Lagrangian nodal velocities with which to move the computational mesh.
  - CAVEAT scheme [Dukowicz et al. (LANL report, 1986)]

Introduction (2)

- However, significant progress has been made recently in solving this problem:
  - 3rd DG scheme [Loubere et al. (IJNF, 2004)]
  - GLACE scheme [Despres and Manzeran (ARMA, 2005)]
  - EUCCLYHD scheme [1]
  - Burton and Shashkov [2]
- A number of cell centred schemes have also now been successfully extended to provide a elastic-plastic flow [1,2].

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[1] Maire P.-H., Abgrall R, Breil J, Loubere R, Rebourcet B, 'A nominally second order cell-centered Lagrangian Scheme for Simulating elastic-plastic flows on two-dimensional unstructured grids', Journal of Computational Physics 235, (2013) 626-665.

[2] Burton DE, Carney TC, Morgan NR, Sambasivan SK, Shashkov MJ, 'A cell-centered Lagrangian Goduno-like method for solid dynamics', Comput. Fluids., **83,** (2013), 33-47.

Introduction (3)

- This talk will focus on the extension of a dual grid cell centred Lagrangian Godunov method [3,4] to provide an elastoplastic flow capability and its performance on strength test problems.
- Numerical results will be compared with results obtained with Compatible FEM SGH [5].

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[3] Barlow AJ, Roe PL, ‘A cell centred Lagrangian Godunov scheme for shock hydrodynamics’. Comput. Fluids., **46,** issue1, (2011), 133-136.

[4] Barlow AJ, ‘A high order cell centred dual grid Lagrangian Godunov scheme’. Comput. Fluids., **83**, (2013), 15-24.

[5] Barlow AJ, ‘A compatible finite element multi-material ALE hydrodynamics algorithm.’, Int. J. Numer. Meth. Fluids (2008); **56**:953-964.

Transient dual grid idea

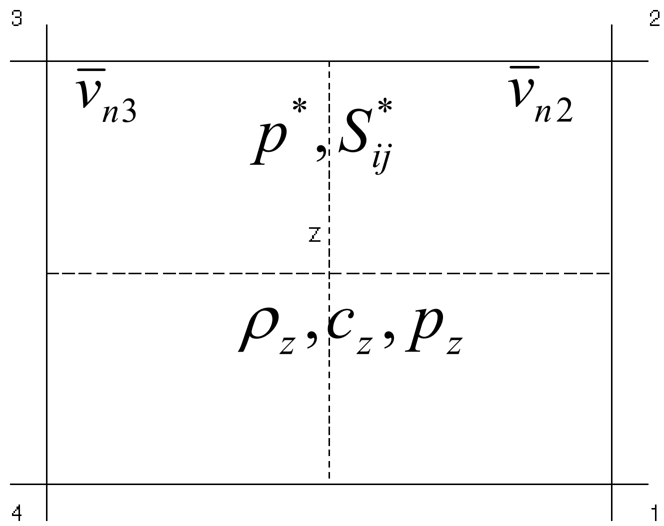
- All variables conserved at element centres.
- Nodal velocities are carried as an additional variable.
- Acceleration of nodes during the time step is calculated by solving an additional momentum equation on transient dual grid.
- Nodal velocities are used to calculate element strain rates.
- Stress deviators calculated as per Wilkins.
- The finite volume update of the conserved element centred variables is performed using fluxing volumes that are consistent with the motion of the nodes.

Nodal acceleration calculation

- P* is required on each of the internal dual grid boundaries in order to solve the nodal momentum equation

$$M_D \left(\frac{d\bar{u}_C}{dt} \right)^{n+\frac{1}{2}} = -\int_{\partial D} \left(p^* - s_{ij}^* \right)^{n+\frac{1}{2}} d\vec{n}$$

- The P* and S* for each median mesh line is obtained by solving a collision Riemann problem using the zonal state variables and the nodal velocities.



Approximate Riemann Solver (1)

- Artificial viscosity methods are often designed to only act normal to the shock front or in the direction of the velocity jump.
- The same idea has been applied here to the acoustic approximate Riemann solver to make it into a simple multi-dimensional solver

$$p^* = \frac{z_{1a} p_{2a} + z_{2a} p_{1a}}{z_{1a} + z_{2a}}$$

$$\bar{q}^* = \frac{z_{1a} z_{2a} (\bar{v}_{2a} - \bar{v}_{1a}) \, | \, \widehat{n}_a . \widehat{a}_a \, |}{z_{1a} + z_{2a}}$$

$$S^*_{ij} = \frac{z_{1a} S_{ij_{2a}} + z_{2a} S_{ij_{1a}}}{z_{1a} + z_{2a}}$$

Approximate Riemann Solver (2)

where

$$\widehat{a}_a = \frac{v_{2a} - v_{1a}}{|\, v_{2a} - v_{1a} \,|}$$

$$z_{1a} = \rho_{1a}\left(\sqrt{c_{1a}^2 + \frac{4\mu}{3\rho}} + \Gamma \,|\, v_{2a} - v_{1a} \,|\right)$$

- This effectively introduces linear and quadratic artificial viscosity like terms as suggested by Dukowicz.
- The sound speed is modified to include the elastic wave speed.

Approximate Riemann Solver (2)

- The zonal and nodal momentum equations can now be written as

$$M_{z,D}\left(\frac{d\overline{u}_{z,D}}{dt}\right)^{n+\frac{1}{2}} = -\int_{\partial D,Z}\left(p^{*}-s_{ij}^{*}\right)^{n+\frac{1}{2}} d\vec{n} - \int_{\partial D,Z}\overline{q}^{\,*}\, ds$$

- The right hand side of these equations can now be viewed as the gathering of forces that are acting on a zone or node.
- From this analogy it is clear how to modify the total energy update to allow for the new approximate Riemann solver.

$$M_{z}\left(\frac{dE_{z}}{dt}\right)^{n+\frac{1}{2}} = -\int_{\partial Z}\left(p^{*}-s_{ij}^{*}\right)^{n+\frac{1}{2}}\overline{u}\ .d\vec{n} - \int_{\partial Z}\left(\overline{q}\right)^{n+\frac{1}{2}}\cdot\overline{u}^{\,n+\frac{1}{2}} ds$$

where u_e is the average velocity of the two nodes defining edge e.

Internal energy update

- Significantly improved results for solids are obtained by determining the internal energy from differences in total energy and kinetic energy rates as suggested by Don Burton.

$$\frac{d\varepsilon}{dt} = \frac{dE}{dt} - \bar{u}.\frac{d\bar{u}}{dt}$$

Time Discretization (1)

$$x^{n+\frac{1}{2}} = x^n + \frac{1}{2} u^n \Delta t \forall nodes$$

$$\rho^{n+\frac{1}{2}} = \frac{M^e}{V^{n+\frac{1}{2}}}$$

$$V^{n+\frac{1}{2}} = V\left(x^{n+\frac{1}{2}}\right) \forall cells$$

Solve Riemann problem for P^{n*}

Define edge velocity u_e as average velocity of two nodes defining edge

$$M_z \frac{dE_z^{n+\frac{1}{2}}}{dt} = -\int_{s_k} (p_e^{n*} - s_{ij}^*) \overline{u}_e^n . d\overline{n}_e^n + \overline{q}_e^{n*} \cdot \overline{u}_e^n ds$$

Predictor total energy update

$$P^{n+\frac{1}{2}} = P\left(\varepsilon^{n+\frac{1}{2}}, \rho^{n+\frac{1}{2}}\right)$$

Equation of State call

$$\varepsilon_z^{n+\frac{1}{2}} = \varepsilon_z^n + \frac{1}{2} \frac{d\varepsilon^n}{dt} dt$$

Time Discretization (2)

Solve Riemann problem for P$^{n+1/2*}$ at cell boundaries

$$M_z \frac{d\overline{u}_z^{n+1}}{dt} = -\int_{s_k} (p_e^* - s_{ij\,e}^*)^{n+\frac{1}{2}} . d\overline{n}_e^{n+\frac{1}{2}*} + \overline{q}_e^{n+\frac{1}{2}*} ds$$

Zonal accelerations

Acceleration calculation - centred pressure for 2nd order accuracy in time

Solve Riemann problem for P$^{n+1/2*}$ at dual mesh boundaries

$$M_p \frac{d\overline{u}_p^{n+1}}{dt} = -\int_{s_k} (p^* - s_{ij}^*)_d^{n+\frac{1}{2}} . d\overline{n}_d^{n+\frac{1}{2}*} + \overline{q}_d^{n+\frac{1}{2}*} ds$$

Nodal accelerations

where $\overline{u} = \frac{1}{2}\left(u^n + u^{n+1}\right)$

$$x^{n+1} = x^n + \overline{u}\Delta t \forall nodes \qquad V^{n+1} = V(x^{n+1}) \forall cells \qquad \rho^{n+1} = \frac{M^e}{V^{n+1}}$$

Corrector total energy update

$$M_z \frac{dE_z^{n+1}}{dt} = -\int_{s_k} p_e^{n+\frac{1}{2}*} \overline{u}_e^{n+\frac{1}{2}} . d\overline{n}_e^{n+\frac{1}{2}} + \overline{q}_e^{n+\frac{1}{2}*} \cdot \overline{u}_e^{n+\frac{1}{2}} ds$$

Define edge velocity u_e as average velocity of two nodes defining edge

$$P^{n+1} = P(\varepsilon^{n+1}, \rho^{n+1})$$

Equation of State call

$$\varepsilon_z^{n+1} = \varepsilon_z^n + \frac{d\varepsilon^{n+\frac{1}{2}}}{dt} dt$$

2^{nd} order extension (1)

- Slope extrapolation is used to determine the velocities, pressures and stress deviators at the cell edges, when a solution to the Riemann problem is required across cell edges.
- Three slopes are calculated in volume coordinates in each isoparametric direction.

$$\frac{\partial \phi_\alpha}{\partial x} = \frac{(\phi_{\alpha+1} - \phi_\alpha)\Delta x_\alpha^2 + (\phi_\alpha - \phi_{\alpha-1})\Delta x_{\alpha+1}^2}{\Delta x_\alpha \Delta x_{\alpha+1}(\Delta x_\alpha + \Delta x_{\alpha+1})}$$

$$\Delta\phi_{\alpha+1} = \frac{\phi_{\alpha+1} - \phi_\alpha}{\Delta x_{\alpha+1}} \qquad \Delta\phi_\alpha = \frac{\phi_\alpha - \phi_{\alpha-1}}{\Delta x_\alpha}$$

2nd order extension (2)

- A van Leer slope limiter is then use to define the slope use for the extrapolation

$$\phi_{\alpha}^{'} = \frac{1}{2}\left(\operatorname{sgn}\left(\Delta\phi_{\alpha}\right) + \operatorname{sgn}\left(\Delta\phi_{\alpha+1}\right)\right)\min\left(\left|\frac{\partial\phi_{\alpha}}{\partial x}\right|, \left|\Delta\phi_{\alpha}\right|, \left|\Delta\phi_{\alpha+1}\right|\right)$$

- A second order approach is also used for the nodal velocities on the dual grid when solving the nodal momentum equation. This appears to be more important for elastoplastic flow problems than pure hydrodynamic problems.
- Currently limiting vectors and tensors component by component. This breaks symmetry and needs to be improved in the future.

Cylindrical Geometry

- An area weighted approach has been taken to extend the method to cylindrical geometry.
- This effectively solves the two momentum equations as for plane geometry case.
- The total energy update is modified to account for the true swept volume associated with each edge in cylindrical geometry by simply using the expression for the average face centred velocity weighted by radius suggested by Pierre-Henri Maire [6]:

$$R_f^c \overline{U}_f^c = \frac{1}{2}\left(\frac{2R_p + R_{p+}}{3} \overline{U}_p + \frac{R_p + 2R_{p+}}{3} \overline{U}_{p+} \right)$$

[6] Maire, P-H, 'A high-order cell centred Lagrangian Godunov scheme for compressible fluid flows in two-dimensional cylindrical geometry', Journal of Computational Physics, 228, (2009), 6882-6915.

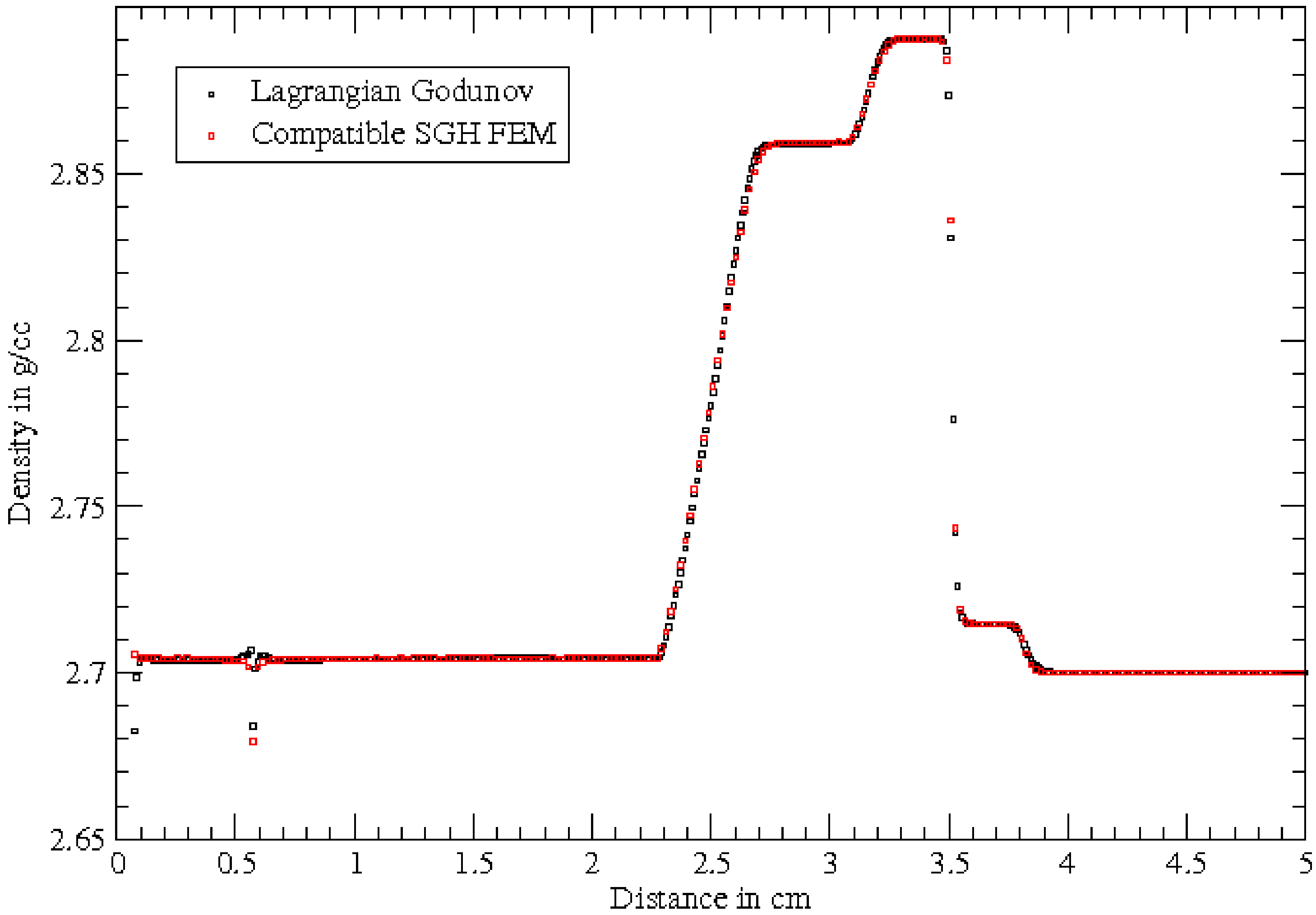
Treatment of free boundaries

- To maintain conservation the momentum and total energy fluxes must be zero across free boundaries.
- It is important to use what ever slope information is available in extrapolating inputs to the Riemann problem for cell adjacent to the free boundaries.

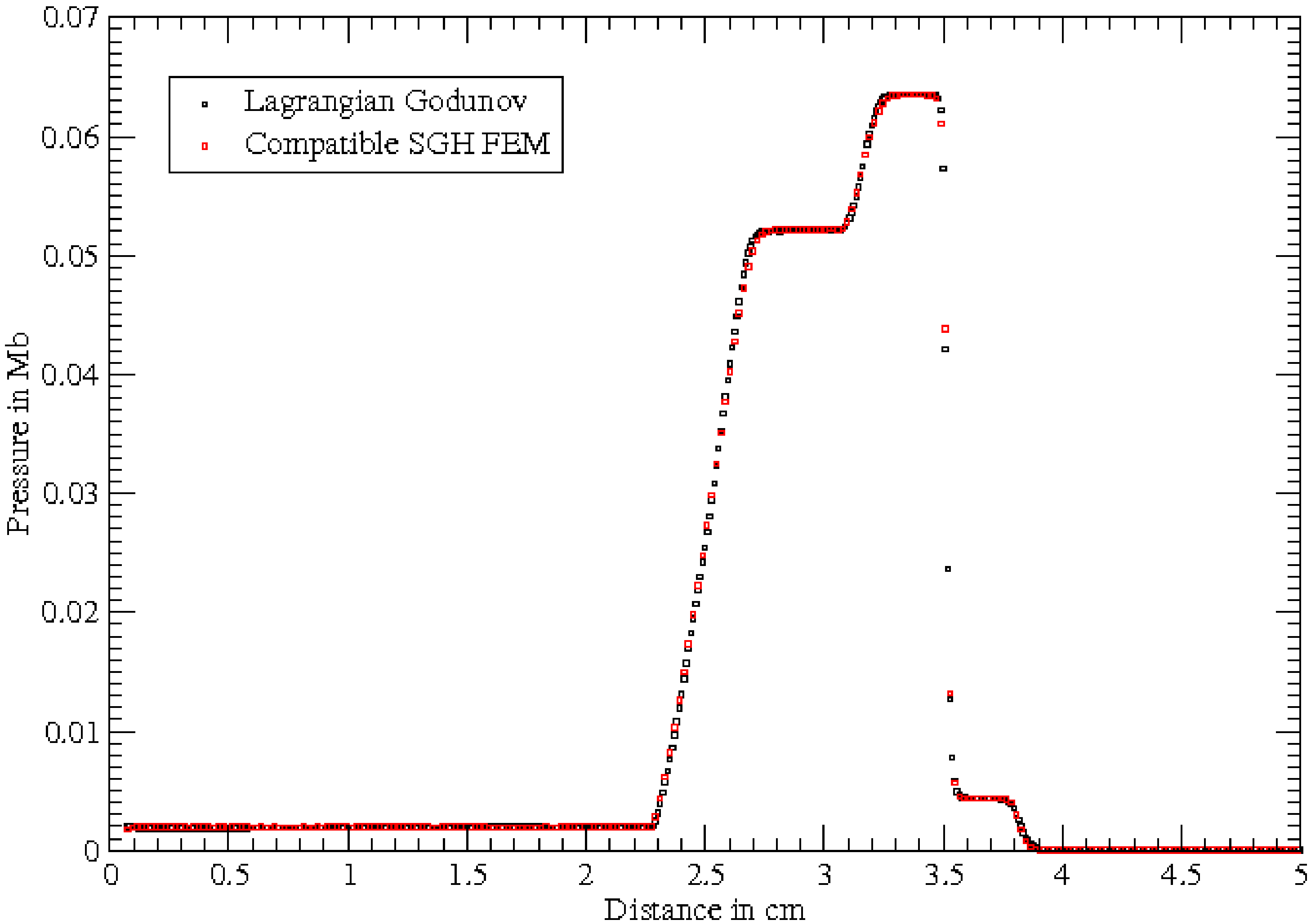
Wilkins Plate impact problem

- 1D Plate impact problem as defined by Wilkins (but using different EoS).
- 5 mm thick Al flyer travelling at 800 m/s strikes an 45 mm thick Al target.
- Constant Yield strength Y=0.003 Mb and constant shear modulus μ=0.276 Mb.
- Initial density ρ=2.7 g/cc. Osbourne EoS.
- 1x500 mesh.

Wilkins plate impact problem t=5.0 μs



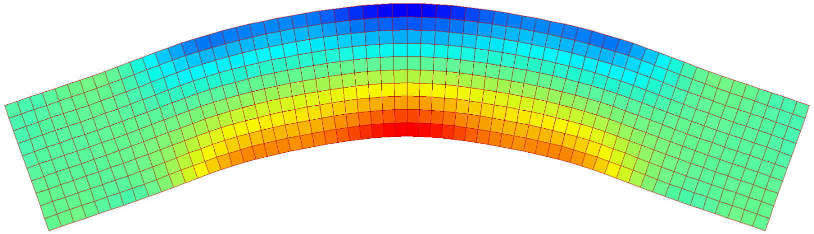
Wilkins plate impact problem t=5.0 μs

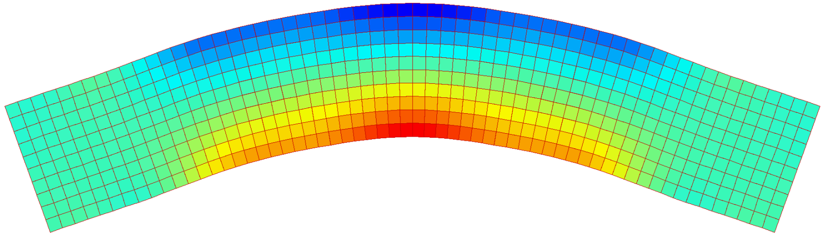


Be Bending Beam

- Be bending beam is a purely elastic test problem
- Consisting of a rectangular plate of infinite extent in z-direction with no supports or constraints with x and y dimensions of 6 and 1 cm respectively.
- Osbourne EoS, constant yield strength Y=1.0 Mb and constant shear modulus μ=1.51 Mb.
- The first flexural mode selected by an initial velocity distribution in y direction.
- 60x40 mesh.

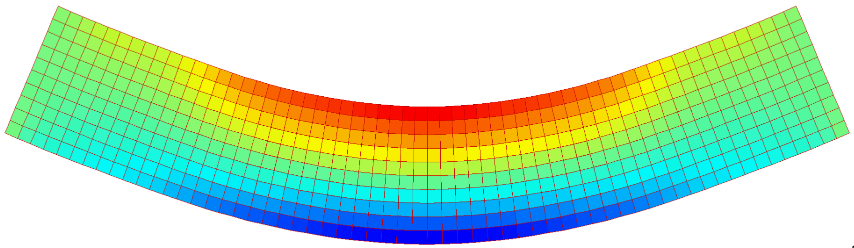
Elastic Bending Beam - Density

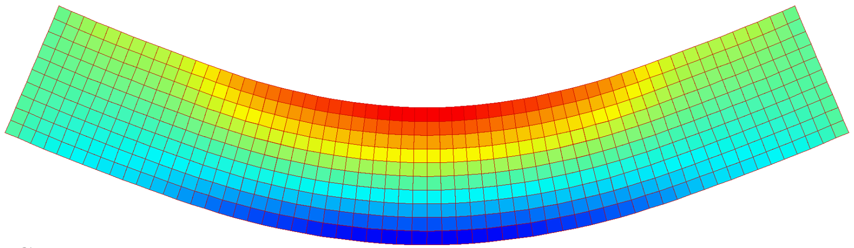




t=7.5 μs

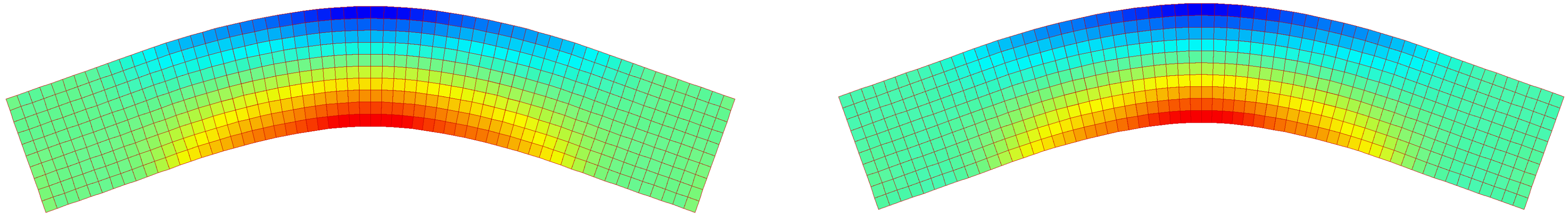
Godunov **SGH**





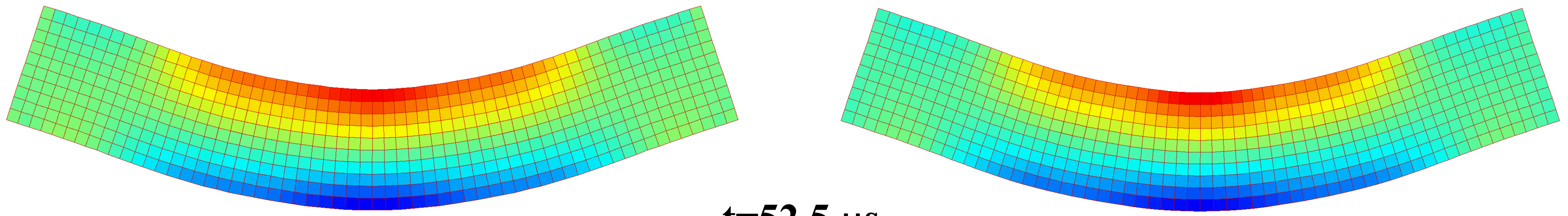
t=22.5 μs

Elastic Bending Beam - Density



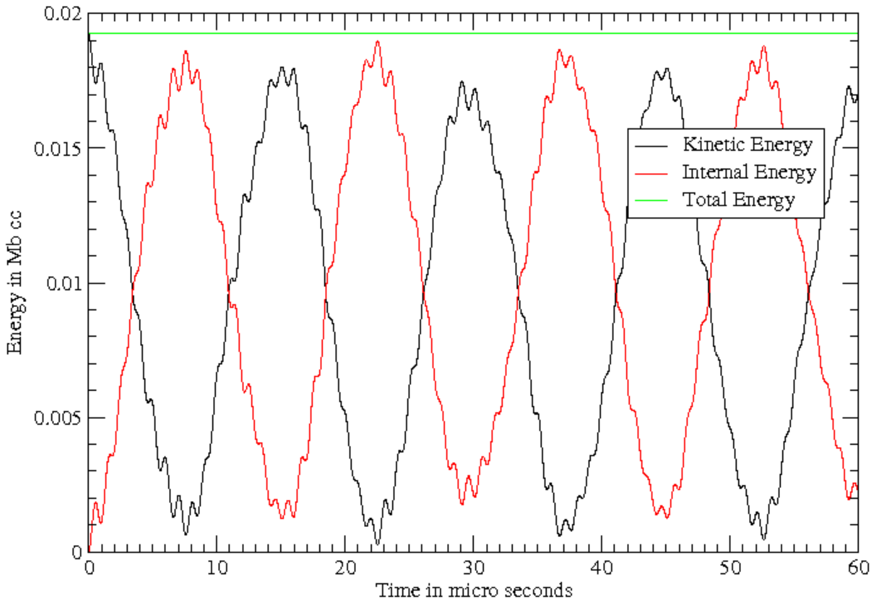
t=37.5 μs

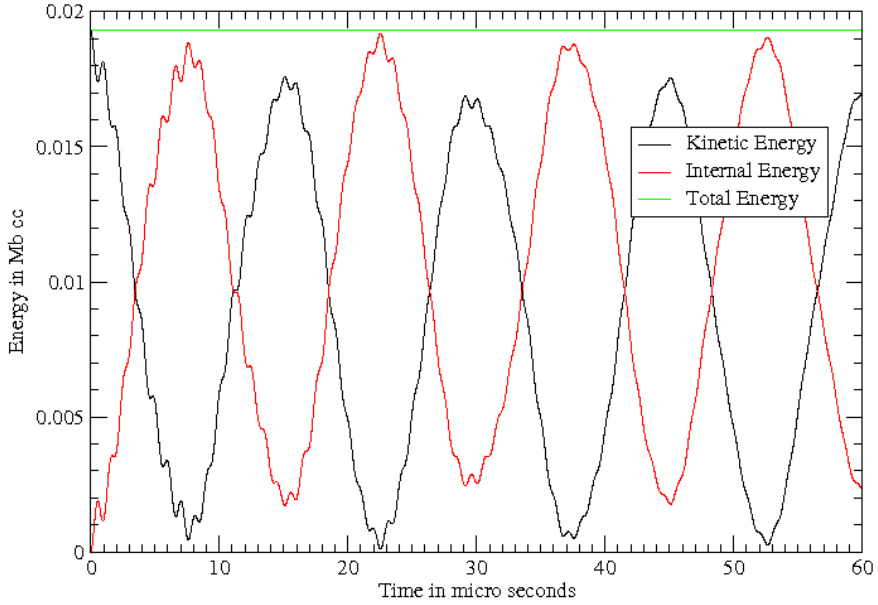
Godunov **SGH**



t=52.5 μs

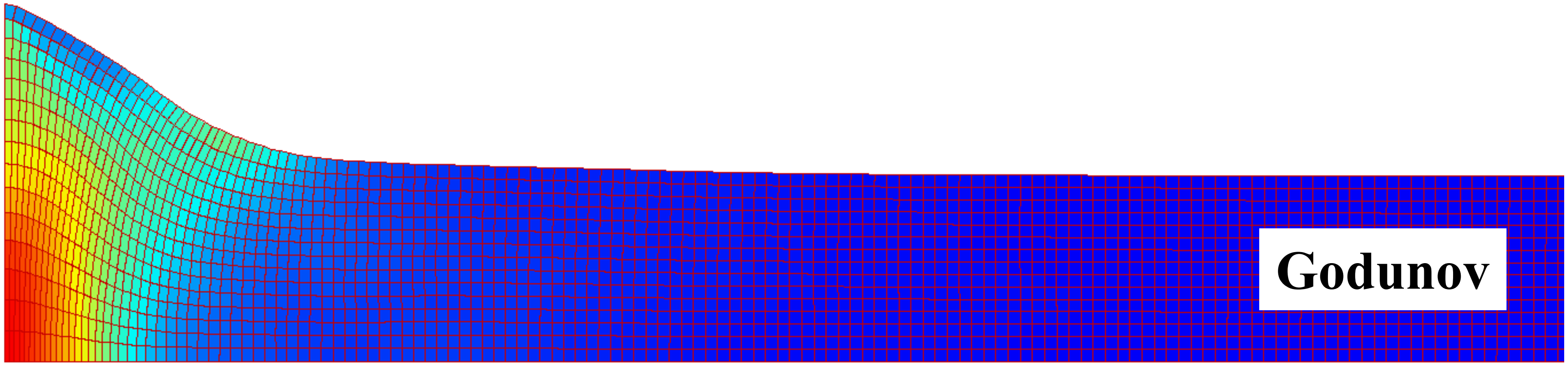
Elastic Bending Beam - Energy



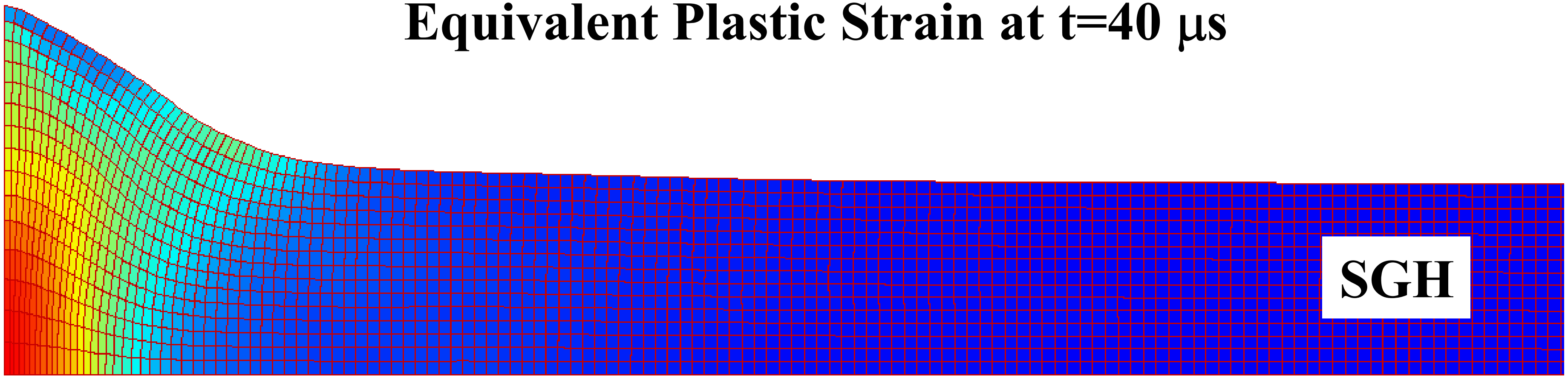


Godunov **SGH**

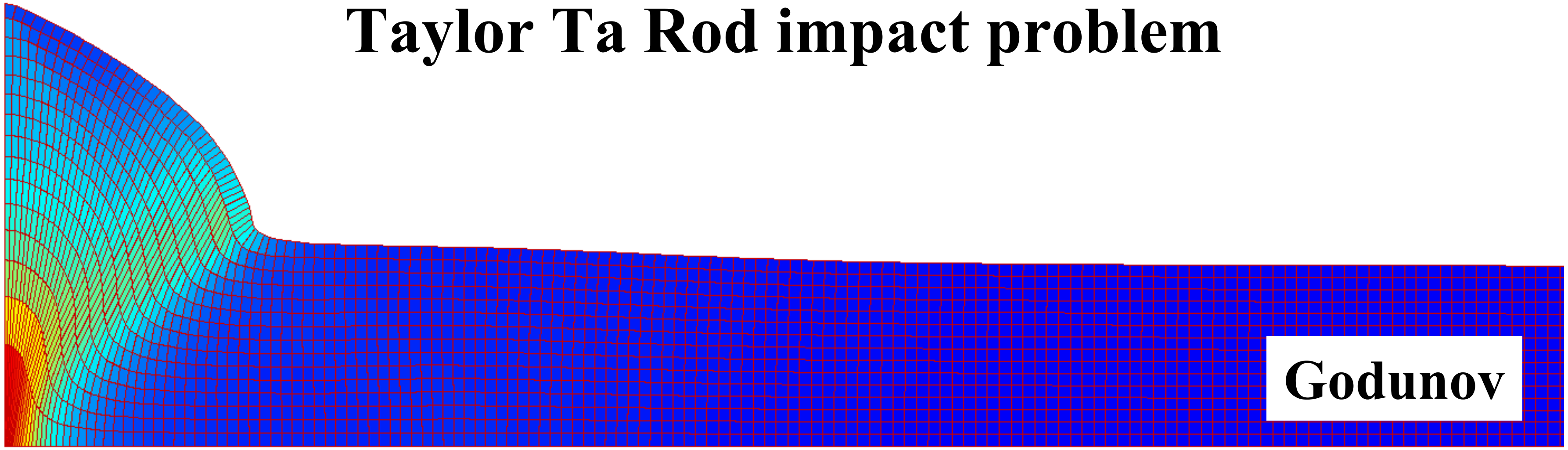
Taylor Ta Rod impact problem 15x200 mesh



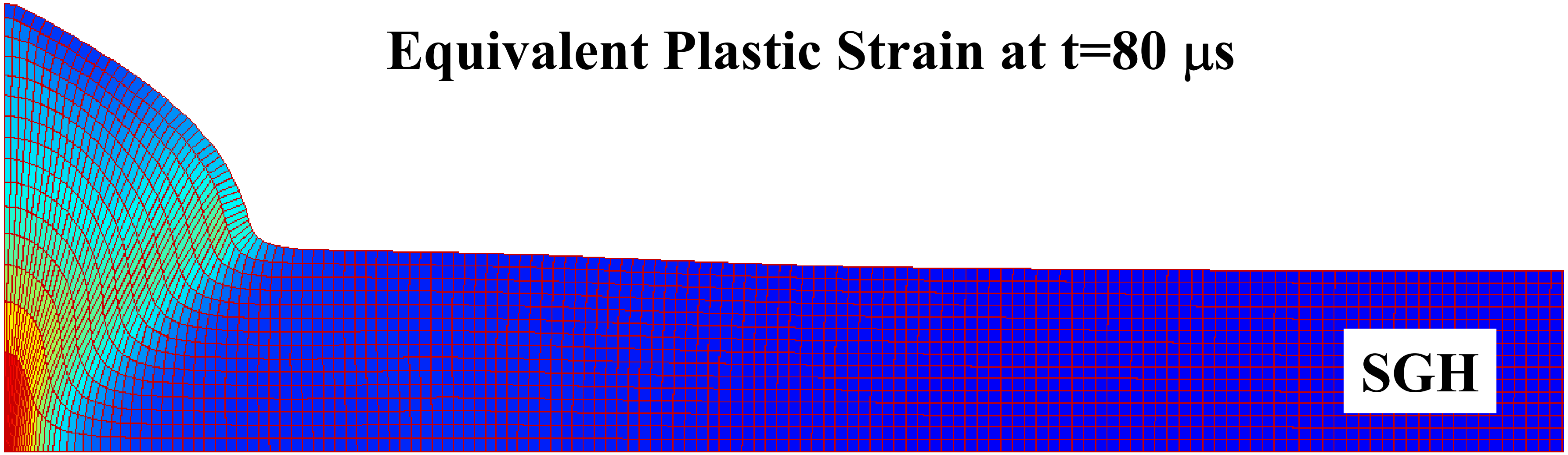
Equivalent Plastic Strain at t=40 μs



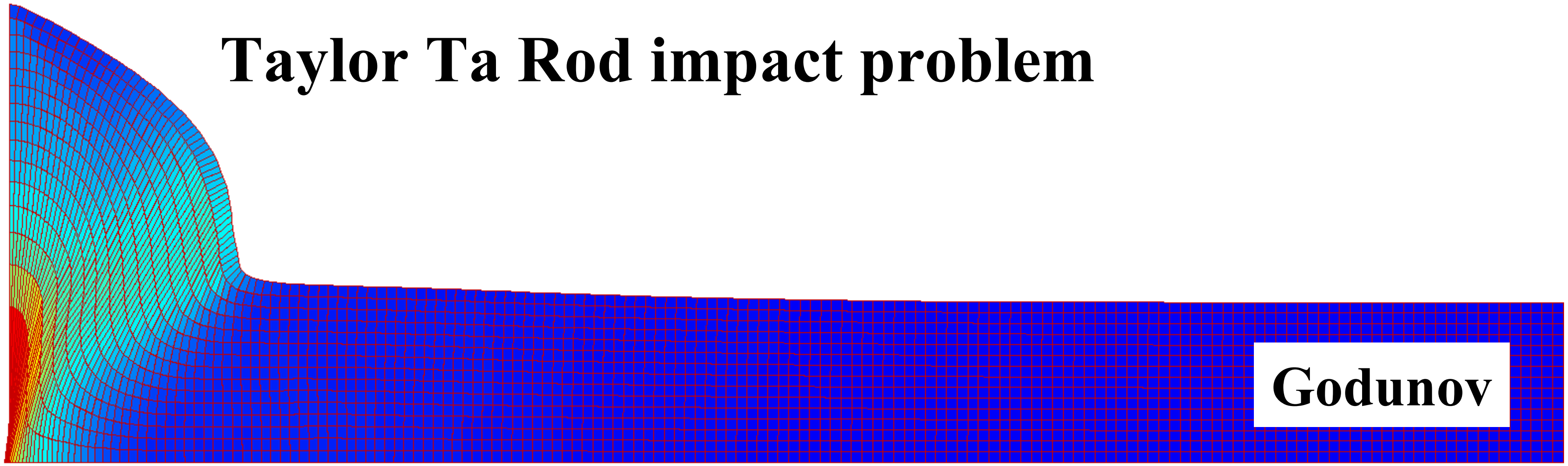
Taylor Ta Rod impact problem



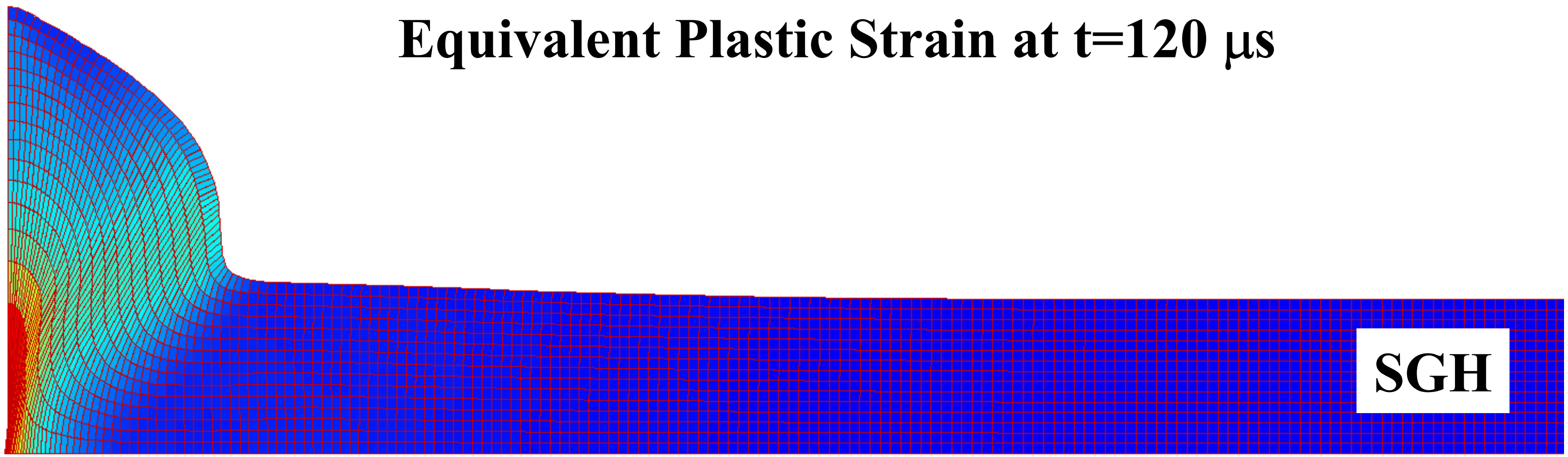
Equivalent Plastic Strain at t=80 μs



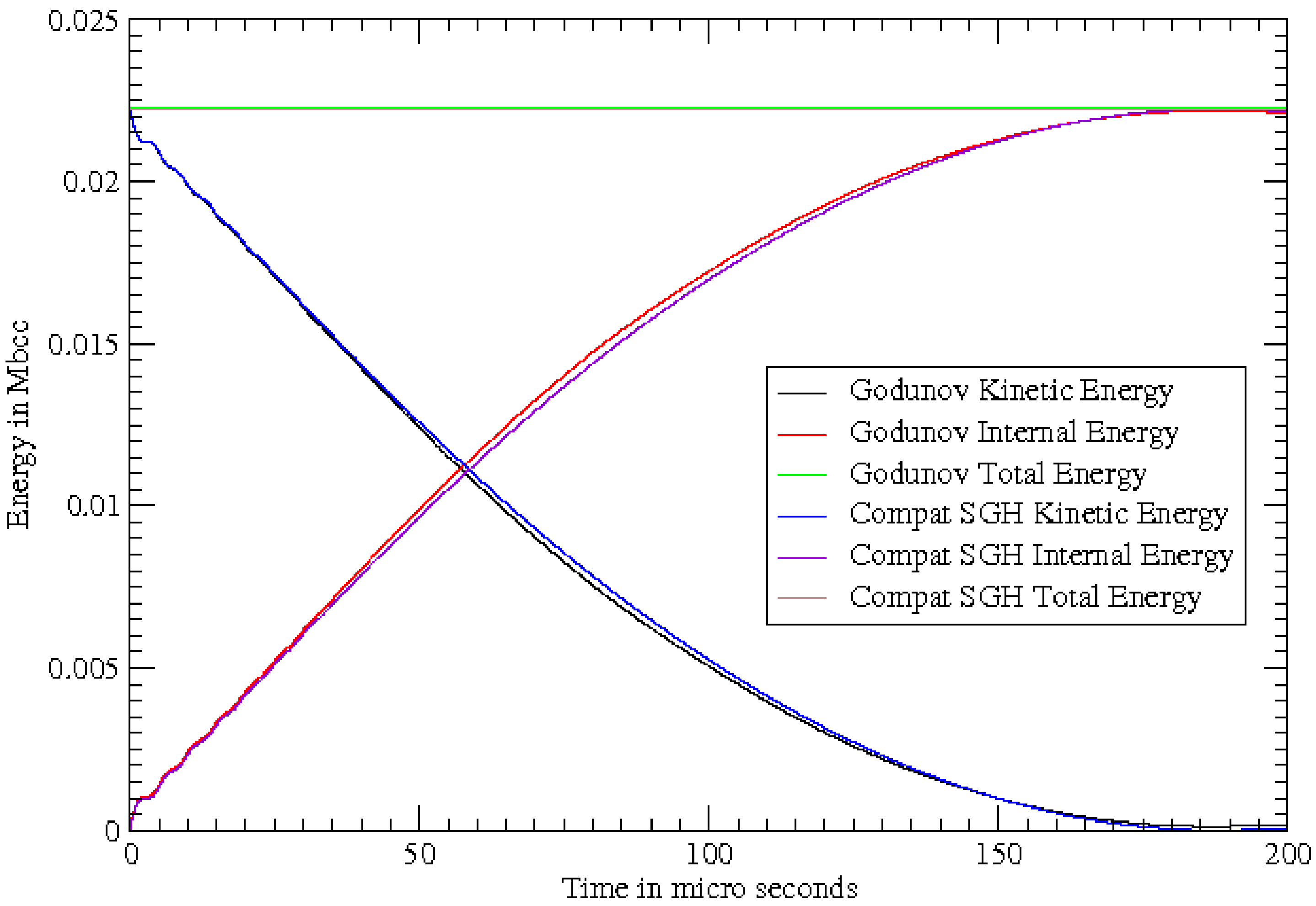
Taylor Ta Rod impact problem



Equivalent Plastic Strain at t=120 μs

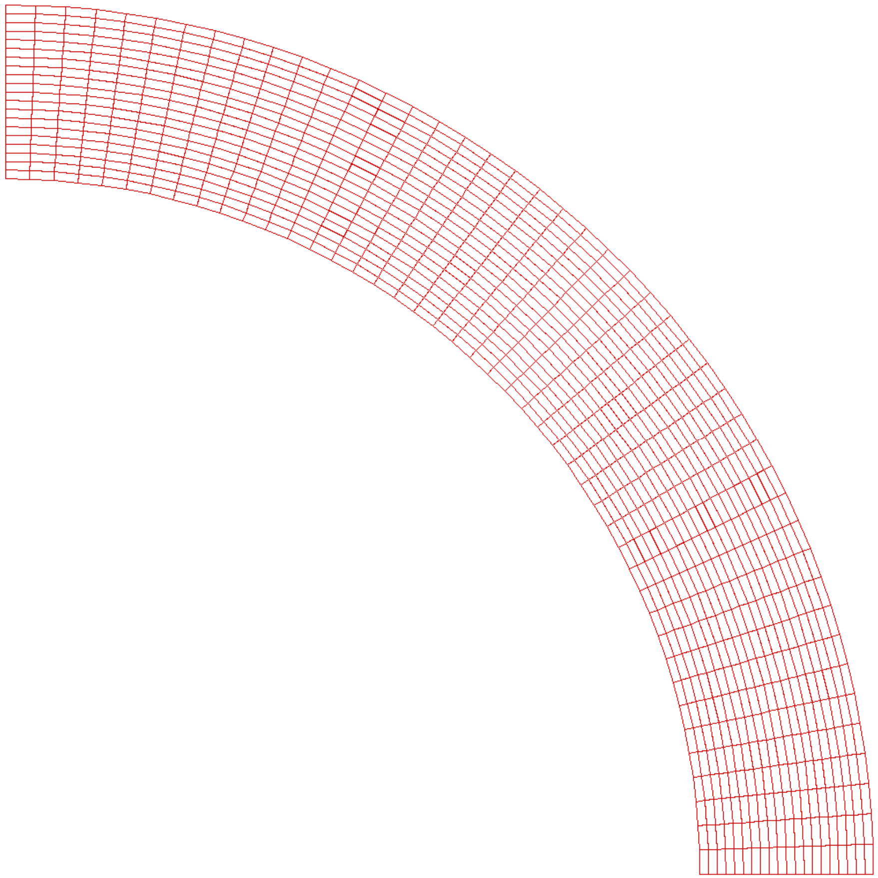


Taylor Ta rod impact problem

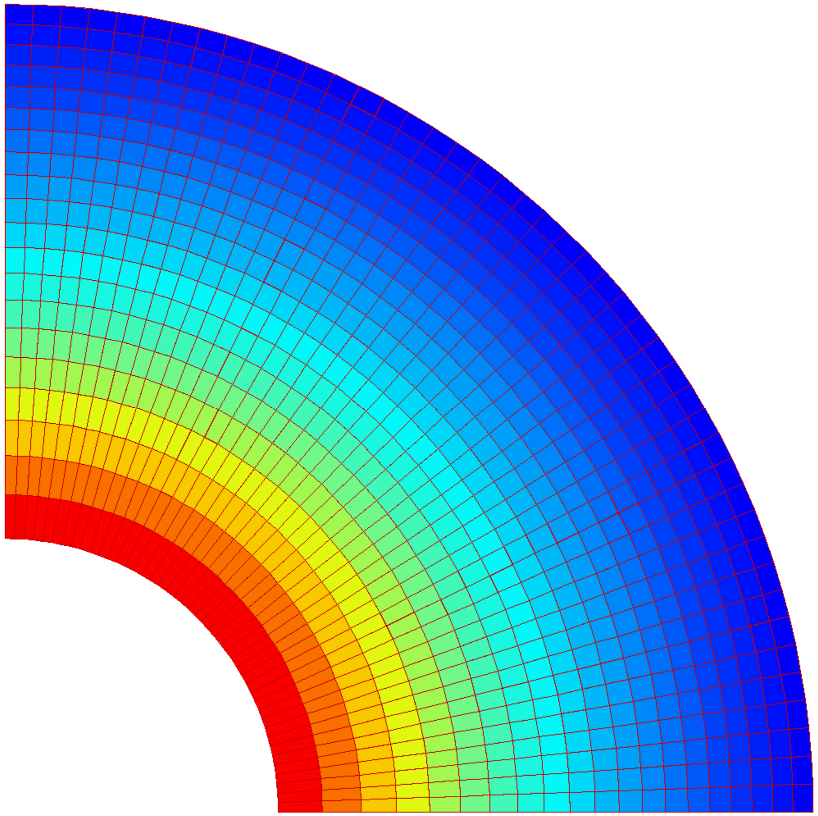


Be Stopping Shell

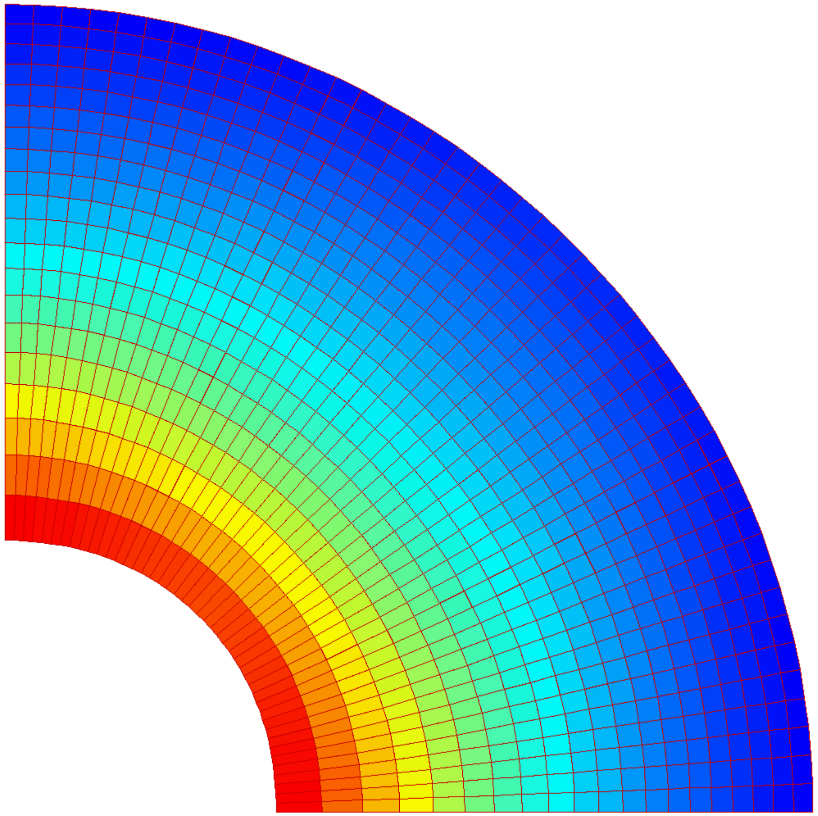
- Be stopping shell, but run in plane geometry (same velocity drive as axisymmetric definition).
- Mesh sensitivity assessed on 3 grids 10x45, 20x45 and 40x45.



Be Stopping Shell – Plastic work at t=145.0 μs

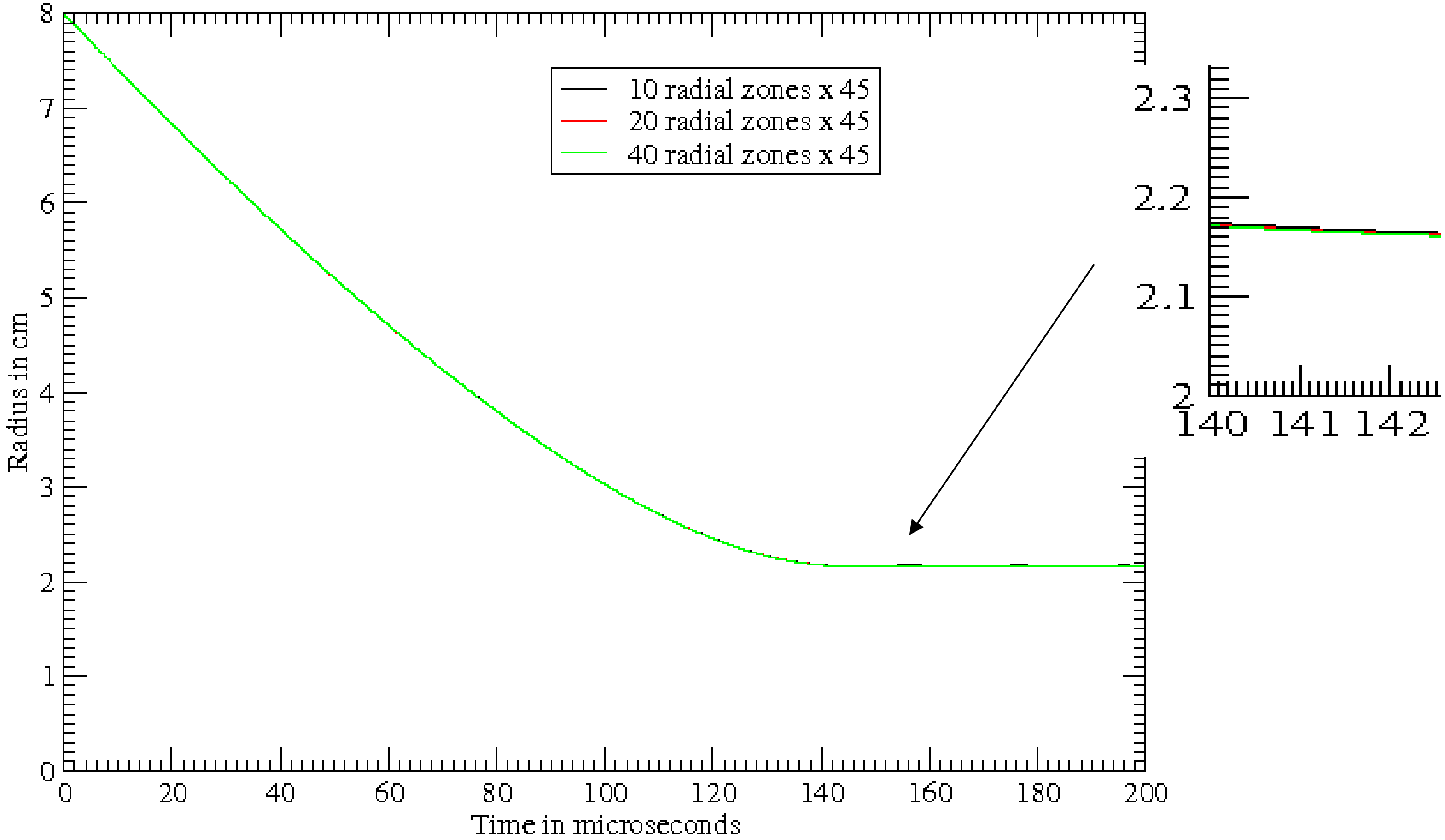


SGH

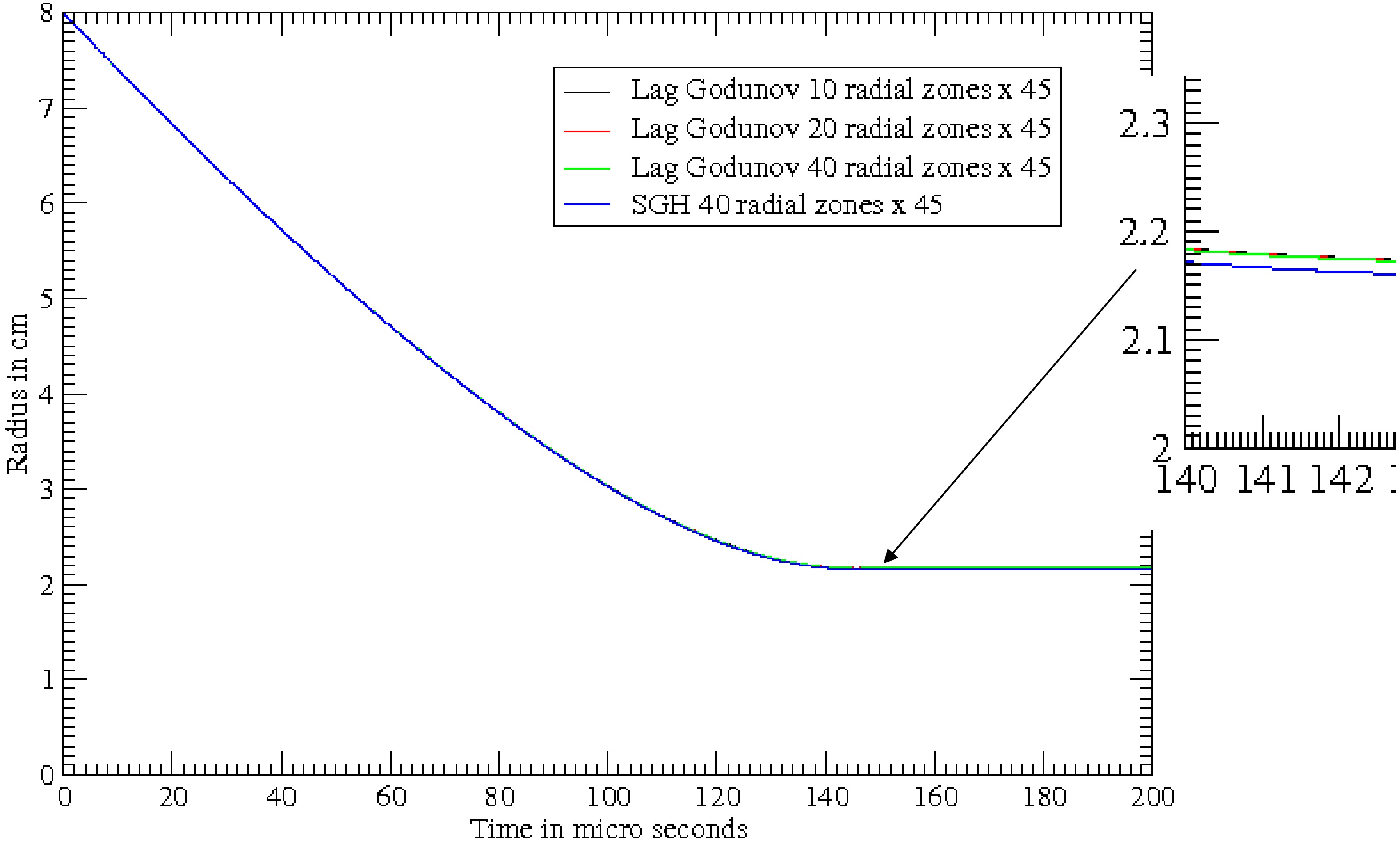


Lagrangian Godunov

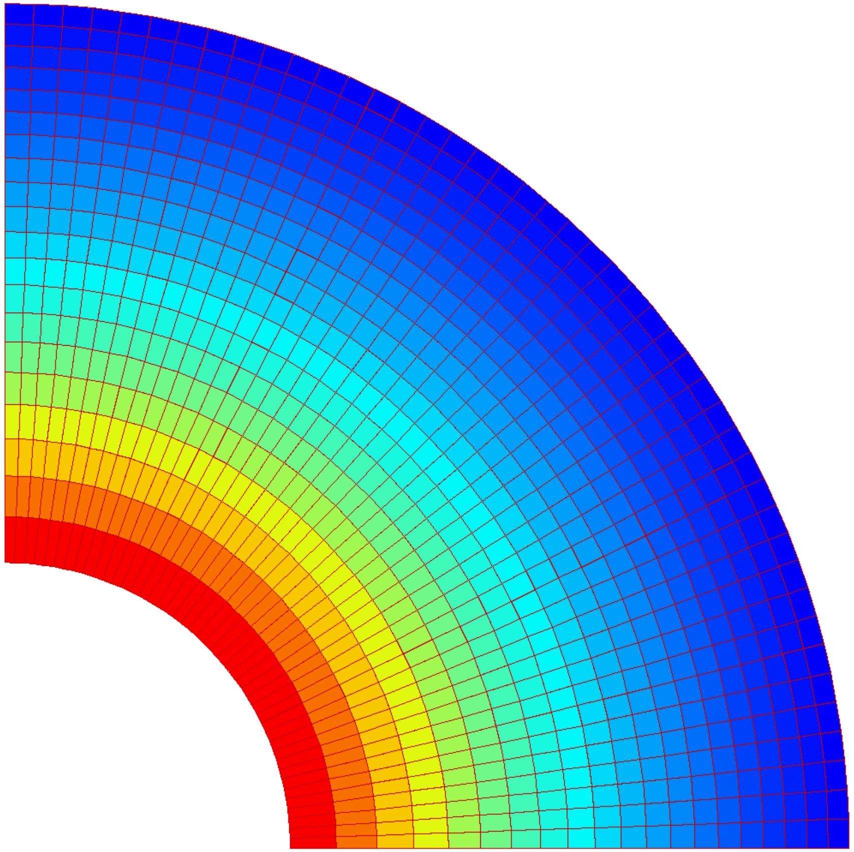
Be Stopping Shell calculated with SGH



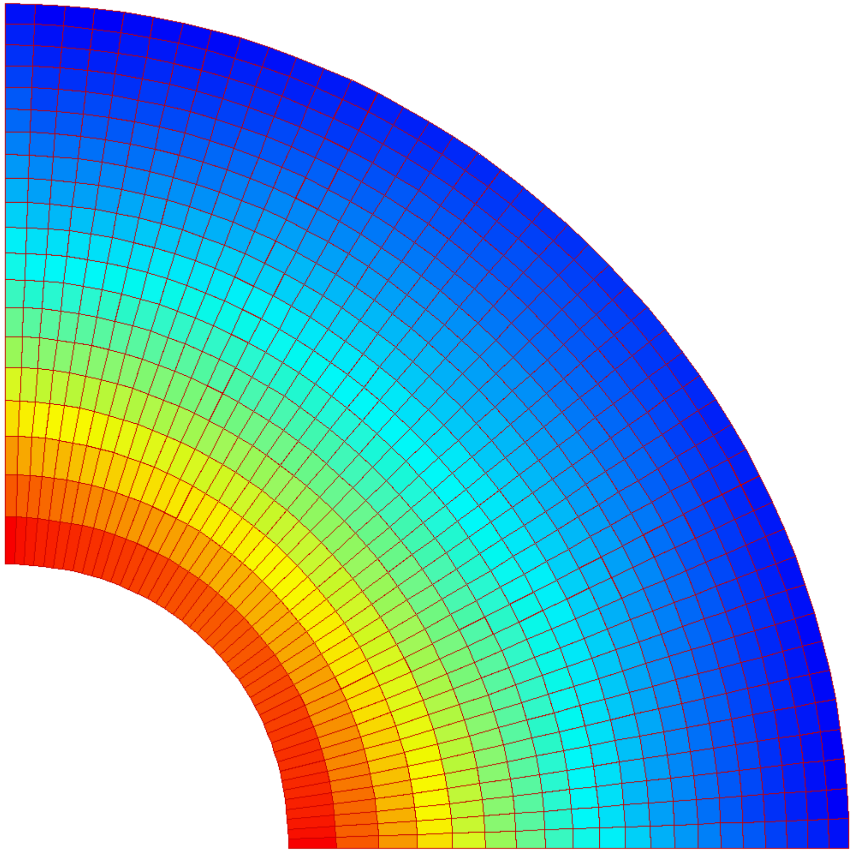
Be Stopping Shell - Lagrangian Godunov



Be Stopping Shell – Internal Energy at t=145.0 μs



SGH



Godunov

Conclusion

- The dual grid cell centred Lagrangian Godunov scheme has been extended to provide elastoplastic flow capability.
- Results have been presented for well known test problems and compared against those obtained with a staggered grid compatible finite element scheme.
- The numerical results are now in good agreement with those of well validated SGH code.
- Symmetry is a little worse due to component by component limiting of the vectors and tensors used in second order extension.

Future Work

- Move to consistent limiting of vector and tensor components to improve symmetry.
- Further verification and validation
  - More rigorous comparisons against analytic solutions
  - Perform axisymmetric strength tests
  - Start to model and compare against relevant experiments
- Extend method to ALE, slide, add additional physics and 3D.